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Note on singular semilinear elliptic equations

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1. Introduction

In this note we study the existence of positive entire solutions for the singular semilinear elliptic equation

$$-\Delta u + c(x)u = p(x)u^{-\gamma}, \qquad x \in \boldsymbol{R}^N,\ N \geq 3,\ \gamma > 0 \tag{1}$$

under the the hypothesis

(H) c and p are locally Hölder continuous functions in $\boldsymbol{R}^N$ with exponent θ, $0 < \theta < 1$, and $c(x) \geq 0$ in $\boldsymbol{R}^N$.

An entire solution of (1) is defined to be a function $u \in C_{\mathrm{loc}}^{2+\theta}(\boldsymbol{R}^N)$ satisfying (1) pointwise in $\boldsymbol{R}^N$.

For the equation (1) with $c(x) \equiv 0$, i.e.,

$$-\Delta u = p(x)u^{-\gamma}, \qquad x \in \boldsymbol{R}^N,\ N \geq 3, \tag{2}$$

Kusano and Swanson [9] proved the existence of a positive entire solution u such that $|x|^{N-2}u(x)$ is bounded above and below as $|x| \to \infty$ under the assumptions that $0 < \gamma < 1$, $p(x) > 0$ in $\boldsymbol{R}^N$ and

$$\int^{\infty} t^{N-1+\gamma(N-2)} p^*(t)\, dt < \infty, \tag{3}$$

where $p^*(t) = \max_{|x|=t} p(x)$. This result was extented afterwards by Dalmasso [2] to cover the case $\gamma \geq 1$.

On the other hand, for the equation (1) with negative γ, it is known that if $-1 < \gamma < 0$, and $p(x)$ satisfies $p(x) > 0$, $\not\equiv 0$ in $\boldsymbol{R}^N$ and

$$\int^{\infty} t p^*(t)\, dt < \infty, \tag{4}$$

then there exists a positive entire solution decaying to 0 at infinity (see e.g. [4], [6], [7] and [10]). However, as far as we are aware, no such result is obtained for the singular type equation (1) under the condition (4).

Our first result, Theorem 1 below, concerns the existence of positive entire solutions of (1) which have uniform positive limits at infnity. In Theorem 2, we show that there exists a decaying entire solution of (1) under the condition