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Component-wise convergence of quasilinearization method for nonlinear boundary value problems

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1. Introduction

The boundary value problem

$$x' = f(t, x), \quad t \in J = [a, b] \tag{1.1}$$

$$F[x] = 0, \tag{1.2}$$

where x and $f(t, x)$ are n dimensional vectors and F is an operator from $C(J)$ into R^n, $C(J)$ is the space of all real n vector functions continuous on J, has been the subject of many recent investigations [1, 2, 5, 8, 13–15, 20]. These, in particular include the existence, uniqueness of the solutions and the convergence of the Picard's and Approximate Picard's iterative schemes. The purpose of this paper is to provide a priori sufficient conditions so that the Quasilinear and Approximate Quasilinear iterative methods for (1.1), (1.2) converge to its unique solution. Since the introduction of the term Quasilinearization by Bellman and Kalaba [7] in the year 1965, also see [4, 6, 9], quasilinear methods have been extensively used to construct the solutions of nonlinear boundary value problems. Therefore, the obtained results in this paper are of theoretical as well as of computational importance. To make the analysis widely applicable all the results are proved component-wise. The significance of such a study for systems is now well recognized from the fact that it enlarges the domain of existence and uniqueness of solutions, weakens the convergence conditions and provides sharper error estimates, e.g., see [1–3, 5, 10–12, 16–20].

2. Notations and Assumptions

Throughout, we shall consider the inequalities between two vectors in R^n component-wise whereas between two $n \times n$ matrices element-wise. The following well known properties of matrices will be used frequently without further mention.

1. For any square matrix A, $\lim_{m \to \infty} A^m = 0$ if and only if $\rho(A) < 1$, where $\rho(A)$