

A study of an identification problem and substitute use of principal component analysis in factor analysis

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0. Introduction

Factor Analysis (FA) is a branch of multivariate statistical analysis which is concerned with the internal relationships of a set of variables. Since Spearman [28] originated FA, it was developed by psychometricians. From 1940's, statisticians have been concerned with FA (see e.g. Lawley [15], Rao [20], Anderson and Rubin [3], Lawley and Maxwell [16], [17]). Factor analysis has been used in many fields of sciences in addition to psychology. Recently program packages applying FA have been developed. However, it may be noted that FA still involves some fundamental problems, and hence an investigation of it is very important.

In an FA model, we assume that an observed p -dimensional vector x follows

$$(0.1) \quad x = \mu + \Lambda f + u,$$

where μ is a mean vector, Λ is a $p \times k$ ($p > k$) factor loading matrix of rank k , f is a common factor vector and u is a unique factor vector. Further, suppose that $E\{f\} = \mathbf{0}$, $E\{u\} = \mathbf{0}$, $E\{uu'\}$ is a diagonal matrix with positive diagonal elements, say Ψ , $E\{fu'\} = O$ and $E\{ff'\} = I$ (a unit matrix). Then, a variance-covariance matrix Σ of x can be decomposed as