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Nonstationary flows of nonsymmetric fluids with thermal convection

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Introduction

The time behaviour of one-component isotropic isothermal fluid is usually described by a Navier-Stokes system (cf., [5], [6], [14]). In contrast to the classical point of view, the balance of angular momentum must be taken into account if the stress tensor is not assumed to be symmetric (cf., [3]). In this case the angular momentum can be decomposed into external and internal parts. The internal part represents the rotational motion of the fluid particles. If the fluid does not accompany any intrinsic motion, the stress tensor is symmetric (cf., [2]). However this situation is relevant for only fluids comprising spherical molecules or those characterized by very low mass density and, in general, the antisymmetric components cannot be neglected.

In this paper we consider the general case and treat the two-term representation of the total stress whose components stand for the scalar equilibrium stress as well as viscous stresses. This setting leads us to an equation of angular momentum balance of the form

$$\omega_t - (c_a + c_d)\Delta\omega - (c_0 + c_d - c_a)\nabla \operatorname{div} \omega + u\cdot\nabla\omega + 4\nu_r\omega = 2\nu_r \operatorname{curl} u + g(\theta), \tag{0.1}$$

where $\omega = (\omega_1, \omega_2, \omega_3)$ denotes the angular velocity vector of the constituent fluid particles, g represents the momentum density of exterior forces, positive constants ν_r, c_0, c_d, c_a are viscosity coefficients associated with the non-symmetricity of the stress tensor. In particular, the rotational viscous motion of the fluid may make both internal and external friction effects. These effects, together with the heat conduction and convection, yield variation of temperature θ. The energy is understood to the the sum of internal energy and kinetic energy. For the viscous flows under consideration, we employ the energy balance equation

$$\theta_t - \kappa\Delta\theta + u\cdot\nabla\theta = \Phi(u, \omega) + h, \tag{0.2}$$

where the positive constant κ is the heat conductivity, h denotes the heat source and Φ is a dissipation function. The dissipation function can be written in the following form which is derived from the energy balance: