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An estimate on the codimension of local isometric imbeddings of compact Lie groups

Dedicated to the memory of Professor Masahisa Adachi

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Introduction

In the previous paper [3], we gave an estimate on the codimension of the Euclidean space into which a Riemannian manifold (M, g) can be locally isometrically or conformally immersed, by using some quantity which is naturally associated with (M, g). In the present paper, we introduce another new quantities of (M, g), and improve the estimate on the codimension based on these newly introduced quantities. The principle of our new method is explained as follows.

Let (M, g) be an n-dimensional Riemannian manifold. We assume that (M, g) is isometrically (or conformally) immersed into the $(n+r)$-dimensional Euclidean space $\boldsymbol{R}^{n+r}$. Let x be a point of M and X be a tangent vector in $T_x M$. We denote by $\mathcal{N}(X)$ the family of linear subspaces W of $T_x M$ satisfying

$$R(Y, Z)X = 0 \qquad \text{for all } Y, Z \in W,$$

where R denotes the curvature tensor field of type (1, 3) at x. We denote by $d(X)$ the maximum dimension of $W \in \mathcal{N}(X)$ and set $p_M(x) = \min d(X)$ $(X \in T_x M)$. Then, by the Gauss equation, or its modified equation for conformal immersions, we have the following inequalities on the codimension r;

$$(*) \qquad \begin{aligned} r &\geq n - p_M(x) && \text{(the isometric case)}, \\ r &\geq n - p_M(x) - 2 && \text{(the conformal case)} \end{aligned}$$

(Proposition 1.1). And using these inequalities, we obtain an estimate on the codimention of isometric or conformal immersions. In fact, we may assert that any open neighborhood of x in M cannot be isometrically (resp. conformally) immersed into the Euclidean space $\boldsymbol{R}^{n+r}$ with $r < n - p_M(x)$ (resp.

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