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Ultimately positive (negative) solutions to a differential inclusion of order n

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1. The aim of this paper is to prove the existence of nonoscillatory solutions with the prescribed asymptotic behaviour of the differential inclusion

$$L_n x(t) \in F(t, x(\varphi(t))), \qquad n > 1, \tag{E}$$

where $L_n x(t)$ is the n-th quasiderivative of $x(t)$ with respect to the continuous functions $a_i(t)\colon J = [t_0, \infty) \to (0, \infty)$, $i = 0, 1, \cdots, n$, $L_0 x(t) = a_0(t)x(t)$, $L_i x(t) = a_i(t)(L_{i-1}x(t))'$, $i = 1, 2, \cdots, n$, $\int_{t_0}^{\infty} a_i^{-1}(t)\,dt = \infty$, $i = 0, 1, \cdots, n-1$, $F(t, x)\colon J \times \mathbf{R} \to \{\text{nonempty convex compact subsets of } \mathbf{R}\}$, $\mathbf{R} = (-\infty, \infty)$ and $\varphi(t)\colon J \to \mathbf{R}$ is a continuous function such that $\lim_{t\to\infty} \varphi(t) = \infty$.

We will use the following notation: $F(t, x)x > (<)0$ means that $yx > (<)0$ for each $y \in F(t, x)$; if $h\colon J \times \mathbf{R} \to \mathbf{R}$, then $F(t, x) \geqq (\leqq) h(t, x)$ means that $y \geqq (\leqq) h(t, x)$ for each $y \in F(t, x)$; if $B \subset \mathbf{R}$, then $|B| = \sup\{|x|\colon x \in B\}$, $\|B\| = \inf\{|x|\colon x \in B\}$. If C is a set, then $cf(C)$ is the set of all convex closed subsets of C.

The basic assumptions on $F(t, x)$ are as follows:

1° $F(t, x)$ is upper semicontinuous on $J \times \mathbf{R}$.

2° $F(t, 0) = \{0\}$ for each $t \in J$.

3° $F(t, x)x < 0$ for each $(t, x) \in J \times \mathbf{R}$, $x \neq 0$;

or

4° $F(t, x)x > 0$ for each $(t, x) \in J \times \mathbf{R}$, $x \neq 0$.

Let $t_0 \leqq b < t < \infty$. Then we denote

$$P_0(t, b) = 1,\ P_i(t, b) = \int_b^t a_1^{-1}(s_1) \int_b^{s_1} a_2^{-1}(s_2) \cdots \int_b^{s_{i-1}} a_i^{-1}(s_i)\,dw_i,$$

$dw_i = ds_i \cdots ds_1$, $i = 1, 2, \cdots, n-1$,

$$Q_n(t, b) = 1,\ Q_j(t, b) = \int_b^t a_{n-1}^{-1}(s_{n-1}) \int_b^{s_{n-1}} a_{n-2}^{-1}(s_{n-2}) \cdots \int_b^{s_{j+1}} a_j^{-1}(s_j)\,dz_j,$$

$dz_j = ds_j \cdots ds_{n-1}$, $j = 1, 2, \cdots, n-1$.

It is easy to see that