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A local Crank-Nicolson method for solving the heat equation

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1. Introduction

A wide range of computations for n-dimensional heat equation $\frac{\partial u}{\partial t} = \alpha \sum_{l=1}^{n} \frac{\partial^2 u}{\partial x_l^2}$ have been extensively investigated today [1], [3], [5], [8], because of their importance in applied sciences. Although the explicit method is computationally simple, it has one serious drawback: The time step δt should be taken to be very small because the process is stable only for $\alpha \sum_{l=1}^{n} \frac{\delta t}{(\delta x_l)^2} \leq \frac{1}{2}$, where δx_l is step size on the space variable. The Crank-Nicolson method has widly been used since it reduces the total volume of calculation and is valid for all small finite value of $r_{x_l} = \frac{\delta t}{(\delta x_l)^2}$. It is however, necessary to solve a large linear system. In this paper, based on the representation of the Trotter product [6], we shall propose a new technique, which does not yield a large linear system by using a splitting of coefficient matrix that is obtained by applying the usual centered difference to the partial differential term in space of the above equation. The proposed method has an explicit form and unconditionally stable. Furthermore, we find that it is superior to the Crank-Nicolson method as is illustrated by numerical examples.

2. The formulation of the local Crank-Nicolson method for one-dimensional problem with the Dirichlet boundary conditions

Let us first consider the following heat equation of (2.1):

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \qquad x \in (0, 1), \ t \geq 0, \tag{2.1}$$

with the initial and the boundary conditions:

$$u(x, 0) = f(x), \qquad x \in (0, 1), \tag{2.2}$$

$$u(0, t) = 0, \qquad t \geq 0, \tag{2.3}$$