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A remark on the global asymptotic stability of a dynamical system modeling two species competition

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1. Introduction

In this paper we shall consider a strongly order preserving semiflow modeling the two species competition in population biology. To motivate the principal result, we consider three examples of competitive dynamical systems in an infinite dimensional space which is a product space. The first example is the model of the unstirred chemostat with equal diffusions [6]

$$\text{(1.1)}'\qquad \begin{aligned} S_t &= dS_{xx} - \frac{m_1 S}{a_1 + S}u - \frac{m_2 S}{a_2 + S}v, \\ u_t &= du_{xx} + \frac{m_1 S}{a_1 + S}u, \qquad 0 < x < 1 \\ v_t &= dv_{xx} + \frac{m_2 S}{a_2 + S}v, \end{aligned}$$

with initial and boundary conditions of the form

$$\begin{aligned} &S_x(0, t) = -S^{(0)}, && u_x(0, t) = v_x(0, t) = 0, \\ &S_x(1, t) + rS(1, t) = 0, && u_x(1, t) + ru(1, t) = 0, \\ &v_x(1, t) + rv(1, t) = 0, \\ &u(x, 0) = u_0(x) \geq 0, && v(x, 0) = v_0(x) \geq 0, \\ &S(x, 0) = S_0(x) \geq 0, && 0 < x < 1, \end{aligned}$$

where d, $S^{(0)}$, r, m_i, a_i, $i = 1, 2$ are positive constants.
From [6], it follows that

$$S(x, t) + u(x, t) + v(x, t) = \varphi(x) + 0(e^{-\alpha t}) \text{ as } t \longrightarrow \infty$$

for some $\alpha > 0$, where $\varphi(x) = S^{(0)}\left(\frac{1 + r}{r} - x\right)$, $0 < x < 1$.

From [10] it suffice to study the dynamics on the ω-limit set, i.e., solutions of