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Oscillatory properties of systems of neutral differential equations

Dedicated to Professor Takaŝi Kusano on his sixtieth birthday

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Abstract. We study oscillatory properties of solutions and existence of nonoscillatory solutions with a power growth at the infinity for the system of differential equations of neutral type

$$\frac{d^{n_i}}{dt^{n_i}}[x_i(t) - a_i(t)x_i(h_i(t))] = p_i(t)f_i(x_{3-i}(g_i(t))), \qquad n_i \in \mathbb{N},\ i = 1, 2.$$

1. Introduction

In this paper we consider systems of neutral differential equations of the form

$$\frac{d^{n_i}}{dt^{n_i}}[x_i(t) - a_i(t)x_i(h_i(t))] = p_i(t)f_i(x_{3-i}(g_i(t))), \qquad n_i \in \mathbb{N},\ i = 1, 2. \tag{S}$$

The following conditions are assumed to hold without further mention:

(a) $a_i, h_i, g_i, p_i\colon \mathbb{R}^+ \to \mathbb{R}$, $f_i\colon \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, are continuous functions;
(b) $h_i(t) \le t$ for $t \in \mathbb{R}^+$, $\lim_{t\to\infty} h_i(t) = \infty$, $\lim_{t\to\infty} g_i(t) = \infty$, $i = 1, 2$;
(c) $zf_i(z) > 0$ for $z \neq 0$.

We put

$$x_i(t) - a_i(t)x_i(h_i(t)) = u_i(t), \qquad i = 1, 2.$$

For $t_0 \ge 0$ denote

$$t_1 = \min\{\inf_{t\ge t_0} h_i(t),\ \inf_{t\ge t_0} g_i(t),\ i = 1, 2\}.$$

A vector function $X = (x_1, x_2)$ is defined to be a solution of system (S) if there exists a $t_0 \ge 0$ such that X is continuous on $[t_1, \infty)$, u_i is n_i times

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