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Travelling wave solutions to a perturbed Korteweg-de Vries equation

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1. Introduction

In fluid dynamics, many authors have tried to explain wave motions on a liquid layer over an inclined plane. To study this problem they have proposed several models which seem to be very interesting also from the mathematical point of view [1], [13]. One of the models is the following partial differential equation derived by Topper and Kawahara [18]:

$$u_t + uu_x + \alpha u_{xx} + \beta u_{xxx} + \gamma u_{xxxx} = 0. \tag{1.1}$$

Here, the wave motion is assumed to depend only on the gradient direction x of the plane. The variable u means the height of the wave at the point x and time t. The physical parameters α, β and γ are all positive. Let us assume that the inclined plane is infinitely long toward the direction of x, that is, $x \in (-\infty, \infty)$. Then (1.1) can be considered as a 1-parameter equation by taking an appropriate scale transformation of u, x and t. For example, (1.1) can be transformed to the following ε-family of equations:

$$u_t + uu_x + u_{xxx} + \varepsilon(u_{xx} + u_{xxxx}) = 0, \qquad t \geq 0, \ -\infty < x < \infty. \tag{1.2}$$

Here ε is a positive parameter. The equation (1.2) is regarded as the Korteweg-de Vries equation when the backward diffusion (u_{xx}) and dissipation (u_{xxxx}) terms are absent [19]. On the other hand, when ε is large it is expected to be close to the Kuramoto-Sivashinsky equation:

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0, \qquad t \geq 0, \ -\infty < x < \infty,$$

which describes chemical turbulence ([10], [11]) or instability of flame front ([17]) and exhibits very complicated patterns. Therefore, one find that ε is a very important parameter, which determines the character of the solutions of the equation. Numerical solutions to (1.2) are shown in Kawahara and Toh [7] with different values of ε. In Figure 1 one of the same numerical solutions to (1.2) with small ε is shown.

In this paper we are mainly concerned with the equation (1.2) when ε is small. At first one might expect that they have similar solutions to those of