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# $J$-groups of the quaternionic spherical space forms

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## 1. Introduction

Let $J(X)$ be the $J$-group of $CW$-complex $X$ of finite dimension. Then by J. F. Adams [2] and D. Quillen [17], it is shown that

$$J(X) = KO(X)/\mathrm{Ker}\, J, \quad \mathrm{Ker}\, J = \textstyle\sum_k (\bigcap_e k^e(\Psi^k - 1)KO(X)), \tag{1.1}$$

where $KO(X)$ is the $KO$-group of $X, J: KO(X) \to J(X)$ is the natural epimorphism and $\Psi^k$ is the Adams operation.

Let $Q_r$ $(r = 2^{m-1} \geqq 2)$ be the generalized quaternion group of order $4r$ given by

$$Q_r = \{x, y: x^r = y^2, xyx = y\},$$

the group generated by two elements $x$ and $y$ with the relations $x^r = y^2$ and $xyx = y$, that is, $Q_r$ is the subgroup of the unit sphere $S^3$ in the quaternion field $H$ generated by the two elements

$$x = \exp(\pi \boldsymbol{i}/r) \quad \text{and} \quad y = \boldsymbol{j}.$$

In this paper, we study the $J$-group of the quaternionic spherical space form:

$$N^n(m) = S^{4n+3}/Q_r \qquad (r = 2^{m-1} \geqq 2),$$

which is the orbit manifold of the unit sphere $S^{4n+3}$ in the quaternion $(n+1)$-space $H^{n+1}$ by the diagonal action of $Q_r$. In the case $m = 2$ and 3, the reduced $J$-group $\tilde{J}(N^n(m))$ is determined by H. Ōshima [15], T. Kobayashi [12], respectively.

Throughout this paper, we identify the orthogonal representation ring $RO(Q_r)$ with the subring $c(RO(Q_r))$ of the unitary represetation ring $R(Q_r)$ through the complexification $c: RO(Q_r) \to R(Q_r)$, since $c$ is a ring monomorphism (cf. (2.1)).

Consider the complex representation $a_0, a_1$ and $b_1$ of $Q_r$ given by

$$\begin{cases} a_0(x) = \ \ \ 1 \\ a_0(y) = -1, \end{cases} \begin{cases} a_1(x) = -1 \\ a_1(y) = \ \ \ 1, \end{cases} b_1(x) = \begin{pmatrix} x & 0 \\ 0 & x^{-1} \end{pmatrix}, \ b_1(y) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$