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A note on G-extensible regularity condition

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0. Introduction

Let X and Y be smooth G-manifolds, where G is a finite group. Then the r-jet bundle $J^r(X, Y)$ is naturally a differentiable G-fibre bundle. Let $J^r_G(X, Y)$ be the subspace of $J^r(X, Y)$ consisting of all the r-jets of "equivariant local maps". Then $J^r_G(X, Y)$ is a G-invariant subspace of $J^r(X, Y)$.

Let $\Omega(X, Y)$ be an open G-subbundle of $J^r(X, Y) \to X$ which is invariant under the natural action by local equivariant diffeomorphisms of X on $J^r(X, Y)$. Then $\Omega(X, Y)$ is called *a natural stable regularity condition.* We say that a map $f: X \to Y$ is *Ω-regular* if $j^r f(X) \subset \Omega(X, Y)$. Now we assume that $\Omega(X, Y)$ be a natural stable regularity condition. We say that $\Omega(X, Y)$ is G-extensible if the following conditions hold: There exists a natural stable regularity condition $\Omega'(X \times \mathbb{R}, Y) \subset J^r(X \times \mathbb{R}, Y)$ (where G acts on $\mathbb{R}$ trivially) such that

$$\begin{cases} \pi(i^*(\Omega'(X \times \mathbb{R}, Y))) = \Omega(X, Y) \\ \pi(i^*(\Omega'(X \times \mathbb{R}, Y) \cap J^r(X \times \mathbb{R}, Y))) = \Omega(X, Y) \cap J^r_G(X, Y), \end{cases}$$

where $\pi: i^*(J^r(X \times \mathbb{R}, Y)) \to J^r(X, Y)$ is the natural projection defined by $\pi(j^r_{(x,0)} f) = j^r_x f \circ i$ for the canonical inclusion $i: X \to X \times \mathbb{R}$. The examples of the G-extensible regularity condition are given in ([2], [3]).

In this paper we will prove the following approximation theorem.

THEOREM 0.1. *Let $\Omega(X, Y)$ be a G-extensible regularity condition, and suppose that there is a continuous equivariant section $\sigma: X \to \Omega(X, Y)$ covering the map $f: X \to Y$. Then f may be fine C^0-approximated by smooth Ω-regular equivariant maps whose r-jets are G-homotopic to σ as sections of $\Omega(X, Y)$.*

This result is an equivariant generalization of the approximation theorem in Appendix of [4]. In [2] we have shown a theorem of homotopy classification on Ω-regular smooth equivariant maps. If we consider an open manifold X, Theorem 1.3 in [2] does not assert that the homotopy class of a proper equivariant map is represented by the jet of an Ω-regular proper smooth equivariant map. However, Theorem 0.1 guarantees this property, so that the theorem refines the previous result in [2].