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Smooth linearization of vector fields near invariant manifolds

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1. Introduction

Linearization of vector fields and diffeomorphisms at a hyperbolic fixed point has been investigated by many authors. In this theory, one tries to locally reduce nonlinear vector fields and diffeomorphisms to linear ones. There are, roughly speaking, two groups of works according to the smoothness class to which the reduction belongs. One group tries to find a smooth (meaning C^r, $r \geq 1$) conjugacy under so-called non-resonance conditions [12] [13] [1] [11]. The second group seek a homeomorphism which conjugates a nonlinear vector field to a linear one [3] [2].

The idea of linearization around fixed points was extended by Pugh and Shub [6] to that around normally hyperbolic invariant manifolds. A similar result was later obtained by Osipenko [4] [5]. In both of the work by Pugh-Shub and that of Osipenko, the conjugacy between nonlinear vector fields (or diffeomorphisms) and linear ones is a homeomorphism. In this regard, their works fall into the second group in the above.

The purpose of this paper is to show that there are situations in which vector fields can be smoothly ($C^r, r > 0$) linearzed in a neighborhood of normally hyperbolic invariant manifolds. The conditions to be placed on the linear part of the vector fields in this paper are considered as a kind of non-resonance conditions. This work therefore falls into the first group in the above. Non-resonance conditions on the linear part of vector fields at fixed points are easy to state because they are algebraic relations between the eigenvalues of a matrix. When one deals with the linear part of vector fields near invariant manifolds, the eigenvalues of matrices are of little use except for special situations, e.g., singularly perturbed vector fields, see [8]. In this paper the non-resonance conditions will be given in terms of a certain relationship among growth and decay rates of solutions of linear differential systems. These can be regarded as gap conditions on the spectra of invariant manifolds in the sense of Sacker and Sell [7].

Although analyses are given to vector fields in this paper, the ideas and techniques employed can readily be modified to treat diffeomorphisms. It is