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Explicit conditions for oscillation of neutral differential systems

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1. Introduction and preliminaries

In the past decade there has been much work on the oscillation of neutral differential equations. Necessary and sufficient conditions via the characteristic equations have been obtained for those with constant coefficients, and various explicit sufficient conditions have been obtained, see [2–6, 8, 9]. For neutral differential systems of first order, O. Arino and I. Györi also gave a necessary and sufficient condition via the characteristic equation, see [1]. Since this condition is not easy to verify, some explicit conditions are needed. But to the best of the authors' knowledge there are very few results so far; here we mention only the results by I. Györi and G. Ladas [7] for a very special system and a weaker definition of oscillation. In this paper we will give some explicit conditions for oscillation of neutral systems under a stronger definition. We will show that even for the scalar case our results for explicit conditions are still the best up to now.

Consider the neutral delay differential system in the form

$$\frac{d^N}{dt^N}[y(t) - Py(t-r)] + \sum_{j=1}^{m} Q_j y(t-\tau_j) = 0 \tag{1.1}$$

where P, Q_j $(j = 1,\dots,m)$ are given $n \times n$ matrices, r, τ_j $(j = 1,\dots,m)$ are nonnegative numbers, $v = \max\{r, \tau_1,\dots,\tau_m\}$ and N is a positive integer.

DEFINITION 1.1. By a solution of (1.1) on $[-v, \infty)$ we mean a function $y \in C([-v, \infty), \mathbf{R}^n)$ such that $y(t) - Py(t-r)$ is N-times continuously differentiable and satisfies (1.1) on $[0, \infty)$.

DEFINITION 1.2. A solution $y = (y_1,\dots,y_n)^T : [-v, \infty) \to \mathbf{R}^n$ of (1.1) is called nonoscillatory if there exists a $t_0 \geq 0$ and $i_0 \in \{1,\dots,n\}$ such that $|y_{i_0}(t)| > 0$, $t \geq t_0$. A solution y of (1.1) is called oscillatory if it is not nonoscillatory. Eq. (1.1) is called oscillatory if all of its solutions are oscillatory.

Note that the definition of oscillation here is much stronger than that in