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On numerical invariants of locally Cohen-Macaulay schemes in $\mathbb{P}^n$

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0. Introduction

Theories of modules with finite local cohomology have been developed during the last decade under the influence of the advance of the theory of Buchsbaum modules (see, e.g., [33], [34], [28]). One of the main purpose of this paper is to give a new perspective of such a theory for equidimensional and locally Cohen-Macaulay schemes in $\mathbb{P}^n$. The main objective of this work is bounding Castelnuovo's regularity and the genus of such schemes. For getting these bounds we describe two approaches: We reduce our problem to the case of points with the Uniform Position property by cutting with sufficiently general linear subspaces, and we apply our results of §2. On the other hand, we study annihilators of local cohomology. Using this approach we will improve Castelnuovo bounds proved, e.g., in [30], [21], [1]. These sharper bounds yield some applications. For example, we solve the problem posed in [31] on p. 367 (see our corollary 3.5). Moreover, both approaches provide a motivation for a new structure which is obtained by introducing (k, ℓ)-Buchsbaum (modules) schemes (see also [14]). Therefore an aim of this paper is also to extend to (k, ℓ)-Buchsbaum schemes some of the results known for arithmetically Buchsbaum subschemes in $\mathbb{P}^n$. For example, this yields new upper bounds even for the genus of curves in $\mathbb{P}^3$ not contained in any surface of degree $< s$.

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1. Notations and preliminary results

Throughout this paper we work over an algebraically closed field of characteristic zero. We set $S = K[X_0, \ldots, X_n]$, $\mathfrak{m} = (X_0, \ldots, X_n) \subset S$ and $\mathbb{P}^n = \mathrm{Proj}\,(S)$. We restrict our attention to subschemes of $\mathbb{P}^n$ which are