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Kuramochi boundaries of infinite networks and extremal problems

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§ 1. Introduction and preliminaries

Discrete potential theory has been developed by several authors, e.g., R. J. Duffin [1] and M. Yamasaki [9] among others, and analogies of various potential theoretic properties of Riemann surfaces have been discussed on infinite networks. For example, the extremal length of a family of paths which tend to the boundary of an infinite network is studied by T. Nakamura and M. Yamasaki [5], the boundary limit of Dirichlet finite functions is investigated by T. Kayano and M. Yamasaki [3] and M. Yamasaki [10] and the extremal problems with respect to the ideal boundary components of an infinite network are discussed by A. Murakami and M. Yamasaki [4].

In this paper, we shall be concerned with the Kuramochi boundaries of infinite networks. In § 3, we give some examples of Kuramochi functions on infinite networks and in § 4, the corresponding Kuramochi boundaries. In the last two sections, we shall study extremal problems related to the Kuramochi boundary; the relation between the extremal distance and the Dirichlet principle related to the Kuramochi boundary in § 5 and the relation between the extremal width and the flow problem with respect to the Kuramochi boundary in § 6.

Let X be a countable set of nodes, Y be a countable set of arcs, K be the node-arc incidence function and r be a positive real function on Y. We assume that the graph $\{X, Y, K\}$ is connected, locally finite and has no self-loop. The quartet $N = \{X, Y, K, r\}$ is called an infinite network. For notation and terminologies concerning infinite networks, we mainly follow [3] and [5].

For a set S denote by $L(S)$ (resp. $L^+(S)$) the set of all real functions (resp. non-negative real functions) on S. For $A \subset X$, by $\varepsilon_A(\in L^+(X))$ we shall mean the characteristic function of A. If $A = \{a\}$, we write ε_a for $\varepsilon_{\{a\}}$. The support of a function f is denoted by Sf.

For $u, v \in L(X)$, we set

$$du(y) = -r(y)^{-1} \sum_{x \in X} K(x, y) u(x), \qquad y \in Y$$