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Integral averaging techniques for the oscillation and nonoscillation of solutions of second order ordinary differential equations

Dedicated to Professor Kusano Takaŝi on his 60th birthday

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1. Introduction

In this paper we consider the second order ordinary differential equation

$$x'' + a(t)f(x) = 0 \tag{1.1}$$

under the following conditions: $a(t)$ is continuous on $[t_0, \infty)$, $t_0 > 0$; $f(x)$ is continuous on $\boldsymbol{R}$, $f'(x)$ exists and is continuous on $\boldsymbol{R} - \{0\}$, and

$$xf(x) > 0 \qquad \text{and} \qquad f'(x) > 0 \qquad \text{for every } x \in \boldsymbol{R} - \{0\}\,.$$

A typical example of (1.1) is the Emden-Fowler equation

$$x'' + a(t)|x|^{\gamma} \operatorname{sgn} x = 0\,, \tag{1.2}$$

where γ is a positive constant.

Our interest here is the problem of oscillation and nonoscillation of solutions of equation (1.1). First we give a new necessary condition for the existence of a nonoscillatory solution $x(t)$ of (1.1), and then, as the contrapositive forms of the result, we establish new oscillation criteria for (1.1). The coefficient $a(t)$ in (1.1) is allowed to take both positive and negative values on any interval $[T, \infty)$, $T \geq t_0$. It is known that, for such an $a(t)$, the integral averages of the integral of $a(t)$ play a crucial role. Let $p \in \boldsymbol{R}$ and $p \geq 1$. There is no need to assume that p is an integer. Then we define $A_p(t)$ by

$$A_p(t) = \frac{1}{t^{p-1}} \int_{t_0}^{t} (t - s)^{p-1} a(s)ds\,, \qquad t \geq t_0\,. \tag{1.3}$$

The important oscillation criteria of Wintner [20]–Hartman [4] (see also [5]) for the linear case and Butler [1] for the nonlinear case involve the asymptotic conditions as $t \to \infty$ of the average function $A_2(t)$. In this paper these oscillation criteria are improved by making use of the general average function $A_p(t)$. Among a number of papers dealing with integral averaging