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On a vector-valued interpolation theoretical proof of the generalized Clarkson inequalities

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Introduction

In [4] Kato gave the generalized Clarkson inequalities by using the Littlewood matrices. Later, Tonge gave in his interesting paper [11] their second proof based on an algebraic structure of these matrices, where the generalized Hausdorff-Young inequality by Williams and Wells [12] is used. He proved them directly for L_p without dealing the scalar case. On the other hand, Maligranda and Persson [8] (see also [9]) recently discussed them in a more generalized form, where an interpolation theoretical treatment is found for the scalar case. (Such a treatment for scalar case is also found in Pietsch's work [10].)

The aim of this paper is, applying complex vector-valued interpolation, to give another direct proof of the generalized Clarkson inequalities. (Unfortunately, 'simple application' to L_p of the argument for the scalar case in Pietsch [10] or Maligranda and Persson [8] stated above does not work well.) Our proof reveals the 'structure' of these inequalities well and it seems to be easily applicable to obtaining these inequalities for some other Banach spaces (cf. the authors [6]). In a special case, our proof may provide one of the most concise proofs of classical Clarkson's inequalities.

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1. Clarkson's and generalized Clarkson's inequalities

In this section, we recall Clarkson's and generalized Clarkson's inequalities, and prepare our tool concerning the complex method of vector-valued interpolation.

Let $L_p = L_p(\Omega, \Sigma, \mu)$, $1 < p < \infty$, be the usual L_p-space on an arbitrary but fixed measure space (Ω, Σ, μ). Let $l_r^n(L_p)$, $1 \leqq r \leqq \infty$, be the space of