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On oscillation of half-linear functional differential equations with deviating arguments

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0. Introduction

This paper is devoted to the study of the oscillatory behavior of half-linear functional differential equations of the type

$$(\mathrm{A}) \qquad (|x'(t)|^{\alpha-1}x'(t))' = \sum_{i=1}^{n} p_i(t)|x(g_i(t))|^{\alpha-1}x(g_i(t)),$$

which can be written as

$$(|x'(t)|^{\alpha}\operatorname{sgn} x'(t))' = \sum_{i=1}^{n} p_i(t)|x(g_i(t))|^{\alpha}\operatorname{sgn} x(g_i(t)),$$

where $\alpha > 0$ is a constant, $p_i: [0, \infty) \to [0, \infty)$ is a continuous function such that $\sup\{p_i(t): t \geq T\} > 0$ for any $T \geq a$, $i = 1, 2, \ldots, n$, and $g_i: [0, \infty) \to R$ is a continuously differentiable function satisfying $g_i'(t) \geq 0$ for $t \geq a$ and $\lim_{t\to\infty} g_i(t) = \infty$, $i = 1, 2, \ldots, n$.

By a solution of (A) we mean a function $x \in C^1[T_x, \infty)$, $T_x \geq a$, which has the property $|x'|^{\alpha-1}x' \in C^1[T_x, \infty)$ and satisfies the equation for all sufficiently large $t \geq T_x$. Our attention will be restricted to those solutions $x(t)$ of (A) which satisfy $\sup\{|x(t)|: t \geq T\} > 0$ for all $T \geq T_x$. It is assumed that (A) does possess such a solution. A solution is said to be oscillatory if it has a sequence of zeros clustering at $t = \infty$; otherwise a solution is said to be nonoscillatory.

The half-linear ordinary differential equation

$$(\mathrm{B}) \qquad (|x'(t)|^{\alpha-1}x'(t))' = p(t)|x(t)|^{\alpha-1}x(t), \qquad p(t) \geq 0,$$

to which (A) reduces when $g_i(t) \equiv t$, $i = 1, 2, \ldots, n$, is nonoscillatory in the sense that all of its solutions are nonoscillatory; see Elbert [1]. However, the presence of at least one deviating argument $g_i(t) \not\equiv t$ in (A) may generate oscillation of some or all of its solutions as the following example shows.

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