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On the primary decomposition of differential ideals of strongly Laskerian rings

Mamoru FURUYA
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Introduction

Let R be a ring and let I be an ideal of R. Let $D = (D_0, D_1, D_2, \dots)$ be a higher derivation of rank m ($1 \leqq m \leqq \infty$) of R such that I is D-differential. On the primary decomposition of such an ideal I, the following is known:

If R is Noetherian and $m = \infty$, then the associated prime ideals of I are D-differential and I can be written as an irredundant intersection $Q_1 \cap \cdots \cap Q_n$ of primary ideals which are D-differential (cf. [4, Theorem 1]).

This result was extended by S. Sato [10] to the case of a set of higher derivations of finite rank. The works of Sato [10] yield the following: Let R be a Noetherian ring and let $\underline{H}$ be a set of higher derivations of rank m ($1 \leqq m \leqq \infty$) of R. Then every $\underline{H}$-differential ideal can be represented as an intersection of a finite number of $\underline{H}$-differential primary ideals of R.

In [10], S. Sato used essentially the assumption that the ring R is Noetherian.

In this paper, we treat the same problem for differential ideals of rings which may be non-Noetherian. The following is obtained: Let R be a ring and let I be an ideal of R. Let $\underline{H}$ be a set of higher derivations of rank m such that I is $\underline{H}$-differential. Then:

(1) If I is a decomposable ideal, $1 \leqq m < \infty$ and $char(R) \neq 0$, then I can be represented as an irredundant intersection of a finite number of $\underline{H}$-differential primary ideals of R.

(2) If R is strongly Laskerian and $1 \leqq m \leqq \infty$, then I can be represented as an irredundant intersection of a finite number of $\underline{H}$-differential primary ideals of R.

1. Preliminaries

All rings in this paper are assumed to be commutative with a unit element. Let R be a ring. A *derivation* of R is an additive endomorphism $d : R \to R$ such that $d(ab) = d(a)b + ad(b)$ for every a, $b \in R$. The set of all derivations of R is denoted by $Der(R)$. Let m be a positive integer. A *higher*