HIROSHIMA MATH. J.
25 (1995), 159–170

Long time behaviour for a diffusion process associated with a porous medium equation

Masaaki INOUE

(Received May 10, 1993)
(Revised September 7, 1993)

0. Introduction

For a given real number $\alpha > 1$, let $\{X(t)\}$ be a d-dimensional diffusion process such that the distribution $P(X(t) \in dx)$ has a density $u(t, x)$ and the generator of $\{X(t)\}$ is $\mathscr{G}_t f(x) = (1/2)u(t, x)^{\alpha-1} \triangle f(x)$, where $\triangle$ is the d-dimensional Laplacian. Then the density function $u = u(t, x)$ has to satisfy

$$(0.1) \qquad (\partial u/\partial t) = (1/2) \triangle (u^{\alpha}), \qquad (t > 0,\ x \in \mathbf{R}^d)$$

in the distribution sense. The equation (0.1) is called a *porous medium equation* ([1]) and the process $\{X(t)\}$ is called a diffusion process associated with (0.1). In the preceding work ([8]), we defined a simple model of many particles flowing through a homogeneous porous medium, and constructed the process $\{X(t)\}$ as a macroscopic limit of the path of each tagged particle and the density u as the same limit of the empirical density of the set of positions of all particles. In this paper, we consider the long time behaviour of the process $\{X(t)\}$ in the following two cases.

Firstly, we consider a random scaling limit. Put

$$K(t) = \int_0^t u(s, X(s))^{\alpha-1} ds,$$

then

$$(0.2) \qquad \lim_{t \to \infty} K(t) = \infty \qquad \text{with probability 1}$$

and

$$(0.3) \qquad \lim_{t \to \infty} E[f(K(t)^{-1/2} X(t))] = \int_{\mathbf{R}^d} f(x)(2\pi)^{-d/2} \exp\{-|x|^2/2\} dx$$

for each $f \in C_b(\mathbf{R}^d \to \mathbf{R})$ (see Theorem 1 in §1).

Secondly, we consider a non-random scaling limit. Put

$$\bar{K}(t) = E[K(t)] \quad \text{and} \quad \beta = 1/(d(\alpha - 1) + 2),$$