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Simple setting for white noise calculus using Bargmann space and Gauss transform

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0. Introduction

Let E_0 be a real separable infinite-dimensional Hilbert space with an inner product $(\cdot, \cdot)_0$ and suppose that we are given a densely defined selfadjoint operator D of E_0 such that D^{-1} is of Hilbert-Schmidt type and $D > 1$. Let $E \subset E_0 \subset E^*$ be a real Gel'fand triplet rigged by the system of norms $\{\| D^p \cdot \|_0; p \in \mathbf{R}\}$ and $H \subset H_0 \subset H^*$ be its complexification. The canonical bilinear forms defined by the pairs of elements $(x, \xi) \in E^* \times E$ and $(z, \eta) \in H^* \times H$ are denoted by $\langle x, \xi \rangle$ and $\langle z, \eta \rangle$, respectively. The functional $C(\xi) = \exp[-\frac{1}{2}\|\xi\|_0^2]$, which is continuous and positive definite in $\xi \in E$, determines a unique probability measure μ on E^* such that

$$\int_{E^*} \exp[\sqrt{-1}\langle x, \xi \rangle]\, d\mu(x) = \exp\left[-\frac{1}{2}\|\xi\|_0^2\right].$$

If $H^* = E^* + \sqrt{-1}\,E^*$ is identified with the product space $E^* \times E^*$, it is possible to define the product measure $\nu = \mu \times \mu$ on H^*. Let $\mathscr{P}(E^*)$ be the space of all polynomials in $\{\langle x, \xi \rangle; \xi \in E\}$ with complex coefficients and $\mathscr{P}(H^*)$ be the space of all polynomials in $\{\langle z, \xi \rangle; \xi \in H\}$, where $x \in E^*$ and $z \in H^*$. Then $\mathscr{P}(E^*)$ is dense in $(L^2) \equiv L^2(E^*, \mu)$. The L^2-closure of $\mathscr{P}(H^*)$ is a proper subspace of $L^2(H^*, \nu)$. This subspace is denoted by $(\mathfrak{F}_0)$. It is called a Bargmann space ([4]).

For $\varphi(x) \in \mathscr{P}(E^*)$, $\varphi(x)$ has a natural analytic continuation $\varphi(w) \in \mathscr{P}(H^*)$ and its restriction to E^* is trivially the original $\varphi(x)$. Thus we can define a map $G: \mathscr{P}(E^*) \to \mathscr{P}(H^*)$ by

$$G\varphi(w) \equiv \int_{E^*} \varphi(x + w/\sqrt{2})\, d\mu(x), \tag{0.1}$$

(ref. Kondrat'ev [17], Hida [10]). This map is called Gauss transform because of its similarity with Gauss transform $\mathscr{G}_t[F]$ of a function $F(v)$ of one real variable v:

$$\mathscr{G}_t[F](u) = \int_{-\infty}^{\infty} F(v + u)(2\pi t)^{-1/2} \exp[-v^2/(2t)]\, dv.$$