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Complex structures on $L(p, q) \times S^1$

Noriaki NAKAGAWA

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0. Introduction

Let M^3 be a compact orientable 3-manifold. Then, $M^3 \times S^1$ has an almost complex structure, because the tangent bundle of M^3 is trivial. However by [8] Theorem 3.1, $M^3 \times S^1$ cannot have any complex structure unless M^3 admits a Seifert fibering structure. Moreover these complex structures are deformation equivalent except the case that M^3 is homeomorphic to a lens space by [8] Theorem 3.2 and [11] Theorems C-1 and C-2. In this note, we determine the deformation types of all the complex structures on the product manifold $L(p, q) \times S^1$. We begin with the precise definition of deformation types or deformation equivalence.

DEFINITION 0.1. ([6] p. 71 Definition 2.9) *When there exists a complex analytic family (M, B, π) such that B is a connected complex manifold and the Jacobian of π has the maximal rank at any point, any two fibers of π are called deformations of each other.*

DEFINITION 0.2. *Complex manifolds X and Y are called deformation equivalent or have the same deformation type if there exists a series of connected complex manifolds X_i for $i = 1, 2, \cdots, n$ such that $X_1 = X$ and $X_n = Y$ and X_{i+1} is a deformation of X_i for $i = 1, \cdots, n-1$.*

REMARK: This definition of deformation equivalence is equivalent to Definition 1.1 in [3].

The purpose of this paper is to prove the following main Theorem 2.1. Let $n(N)$ denote the number of deformation types of the complex manifolds which are diffeomorphic to the manifold N.

THEOREM 2.1. *Let p and q be positive integers with $p > 1$ and $(p, q) = 1$ and $L(p, q)$ a 3-dimensional lens space. Then,*

$$n(L(p, q) \times S^1) = \begin{cases} 1 & \text{if } q^2 \equiv -1 \pmod p \\ 2 & \text{if } q^2 \not\equiv -1 \pmod p \end{cases}$$

The latter case is characterized as the case that $L(p, q)$ and $L(p, -q)$ are