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On the irreducible components of the solutions of Matsuo's differential equations

Dedicated to Professor Kiyosato Okamoto on his sixtieth birthday

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0. Introduction

Studying the Knizhnik-Zamolodchikov equation in conformal field theory, Matsuo found a new system of differential equations of first order for a function taking values in the group algebra $\mathbf{C}[W]$ of the Weyl group W associated with an arbitrary root system in [4]. His system is equivalent to the system of the differential equations given by Heckman and Opdam which is a deformation of the system satisfied by the zonal spherical function of the Riemannian symmetric space G/K of non compact type ([4] Theorem 5.4.1).

Let Φ be a solution of Matsuo's equations (see (1.1)). $\hat{W}$ denotes the set of the equivalence classes of the irreducible representations of W. For $\delta \in \hat{W}$ let E_δ be a representation space of δ and $n_\delta = \dim E_\delta$. Then $\mathbf{C}[W] = \sum_{\delta \in \hat{W}} \mathbf{C}[W]_\delta$, where $\mathbf{C}[W]_\delta = \bigoplus_{i=1}^{n_\delta} E_{\delta,i}$ and $E_{\delta,i}$ is equivalent to E_δ $(1 \leq i \leq n_\delta)$. Let δ_0 be the trivial representation and Φ_0 be the $\mathbf{C}[W]_{\delta_0}$-component of Φ. The Correspondence $\Phi \to \Phi_{\delta_0}$ gives the equivalence of the above two systems.

For $\delta \in \hat{W}$ We consider the other $\mathbf{C}[W]_\delta$-components Φ_δ of Φ. In this paper we obtain a system of differential equations satisfied by Φ_δ.

1. Preliminaries

Let E be an n-Euclidean space with the inner product $(\ ,\ )$ and E^* be the dual space of E. For $\alpha \in E$ with $\alpha \neq 0$ put $\alpha^\vee = 2(\alpha, \alpha)^{-1}\alpha$ and denote $s_\alpha(\lambda) = \lambda - (\lambda, \alpha^\vee)\alpha$ for the orthogonal reflection in the hyperplane perpendicular to α $(\lambda \in E)$. Let $\Sigma \subset E$ be a root system with rank $(\Sigma) = \dim E = n$. Fix a system of positive roots Σ^+ in Σ. Furthermore we put $\Sigma_0 = \{\alpha \in \Sigma;\ \alpha \notin 2\Sigma\}$ and $\Sigma_0^+ = \Sigma_0 \cap \Sigma^+$. Let W be the Weyl group and $\mathbf{C}[W]$ be the group algebra of W. Put $\mathfrak{a} = E^*$, $\mathfrak{h} = E^* \oplus iE^*$. The inner product in E and the reflections can be extended to $\mathfrak{h}^*$ naturally. We identify $\mathfrak{h}^*$ with $\mathfrak{h}$ via the inner product $(\ ,\ )$: