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Nonhomogeneity of Picard dimensions for negative radial densities

Dedicated to Professor Fumi-Yuki Maeda on his 60th birthday

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Consider the punctured open unit ball $\{0 < |x| < 1\}$ in the punctured Euclidean m-space $R^m \setminus \{0\}$ $(m \geq 2)$ in which we regard the origin $x = 0$ as an ideal boundary component of $R^m \setminus \{0\}$. For each s in $(0, 1]$ we set $U_s = \{0 < |x| < s\}$, which is also an ideal boundary neighbourhood of the ideal boundary component $x = 0$ in $R^m \setminus \{0\}$, so that Γ_s: $|x| = s$ is the relative boundary of U_s and the relative closure $\bar{U}_s$ of U_s in $R^m \setminus \{0\}$ is $U_s \cup \Gamma_s$. We set $U_1 = U$ and $\Gamma_1 = \Gamma$. A *density* $P(x)$ on U_s is a locally Hölder continuous function defined on $\bar{U}_s$. Consider the time independent Schrödinger equation

$$L_P u(x) \equiv -\Delta u(x) + P(x)u(x) = 0 \tag{1}$$

defined on $\bar{U}_s$, where Δ is the Laplacian $\Delta = \sum_{i=1}^{m} \partial^2/\partial x_i^2$. We are interested in the class $P(U_s, P)$ of nonnegative solutions of (1) in U_s with vanishing boundary values on Γ_s. Let $r\omega$ be the polar coordinate expression of x, where $r = |x|$ and $\omega = (x/|x|) \in \Gamma$. We set

$$l(u) \equiv -\frac{s}{\omega_m} \int_\Gamma \left[\frac{\partial}{\partial r} u(r\omega) \right]_{r=s} d\omega,$$

where $d\omega$ is the area element on Γ, ω_m the area of Γ and $\partial/\partial r$ the outer normal derivative on Γ_s considered in $\bar{U}_s$. It is convenient to consider the subclass $P_1(U_s, P) \equiv \{u \in P(U_s, P); l(u) = 1\}$. Since $P_1(U_s, P)$ is convex, we can consider the set $ex.P_1(U_s, P)$ of extreme points of $P_1(U_s, P)$ and the cardinal number $\#(ex.P_1(U_s, P))$ of $ex.P_1(U_s, P)$ which will be referred to as the *Picard dimension* of (U_s, P) at $x = 0$, $\dim(U_s, P)$ in notation:

$$\dim(U_s, P) = \#(ex.P_1(U_s, P)).$$

There exists a t in $(0, 1]$ such that $\dim P(U_s, P) = \dim P(U_t, P)$ for any s in $(0, t]$ ([8], [7], [9]). Hence we can define the *Picard dimension* of P at $x = 0$, $\dim P$ in notation, by

$$\dim P = \lim_{s \downarrow 0} \dim(U_s, P).$$