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Taylor expansion of Riesz potentials

Dedicated to Professor Fumi-Yuki Maeda on the occasion of his sixtieth birthday

Tetsu SHIMOMURA and Yoshihiro MIZUTA
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Abstract: This paper deals with Riesz potentials $U_\alpha f(x) = \int |x-y|^{\alpha-n} f(y)dy$ of functions f satisfying Orlicz condition with weight ω in the form:

$$\int \Phi_p(|f(y)|)\omega(|y|)dy < \infty.$$

We are mainly concerned with the case when $\Phi_p(r)/r^p$, $p > 1$, is nondecreasing and $\omega(r)$ is of the form r^β, $-n < \beta \leq \alpha p - n$. Letting ℓ be the integer such that $\ell \leq \alpha - (n+\beta)/p < \ell + 1$, we examine when

$$\lim_{x \to 0, x \in R^n - E} [\kappa(|x|)]^{-1}[U_\alpha f(x) - P(x)] = 0$$

holds for an exceptional set E, a weight function κ and a polynomial P of degree at most ℓ.

1. Introduction

For $0 < \alpha < n$ and a nonnegative measurable function f on R^n, we define $U_\alpha f$ by

$$U_\alpha f(x) = \int_{R^n} |x-y|^{\alpha-n} f(y)dy.$$

Here it is natural to assume that $U_\alpha f \not\equiv \infty$, which is equivalent to

$$\int_{R^n} (1+|y|)^{\alpha-n} f(y)dy < \infty. \tag{1.1}$$

To obtain general results, we treat functions f satisfying a condition of the form:

$$\int_{R^n} \Phi_p(f(y))\omega(|y|)dy < \infty. \tag{1.2}$$