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Oscillations of half-linear second order differential equations

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Abstract. Some oscillation criteria are given for the half-linear second order differential equation

$$[\Phi(u'(t))]' + c(t)\Phi(u(t)) = 0,$$

where $\Phi: \mathbb{R} \to \mathbb{R}$ is defined by $\Phi(s) = |s|^{p-2}s$ with a fixed number $p > 1$ and $c \in C([0, \infty), \mathbb{R})$. These results improve Willett's results.

1. Introduction

Define $\Phi: \mathbb{R} \to \mathbb{R}$ by $\Phi(s) = |s|^{p-2}s$, where $p > 1$ is a given number. Consider the half-linear second order differential equation

$$\text{(E)} \qquad [\Phi(u'(t))]' + c(t)\Phi(u(t)) = 0,$$

where $c(t)$ is a continuous function on $[0, \infty)$. We observe that if $p = 2$, then equation (E) reduces to the linear equation

$$\text{(E}_1\text{)} \qquad u''(t) + c(t)u(t) = 0.$$

Let u be a positive solution of (E). If $u' > 0$ or $u' < 0$, then (E) reduces to the following Euler-Lagrange equations:

$$\frac{d}{dt}[u'(t)]^{p-1} + c(t)u^{p-1}(t) = 0$$

or

$$\frac{d}{dt}[-u'(t)]^{p-1} - c(t)u^{p-1}(t) = 0,$$

respectively.

By a solution of (E) we mean a function $u \in C^1[0, \infty)$ such that $\Phi(u') \in C^1[0, \infty)$, satisfying equation (E). Elbert [1] established the existence, uniqueness and extension to $[0, \infty)$ of solutions to the initial value problem for (E). We say that a nontrivial solution u of (E) is oscillatory if for any $N > 0$ there exists $t > N$ such that $u(t) = 0$, otherwise, it is nonoscillatory.