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Tessellation automata on free groups

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Introduction

Tessellation automata on finitely generated free groups are investigated. Given a finitely generated free group G, we can construct hyperbolic tessellation on its Cayley graph, which is a tree. The vertex set of the graph is G itself. On each vertex we place an automaton with a finite set of states Q. Each of these automata is influenced by its neighbors, the number of which equals twice the number of generators of the free group G. With these local interactions we construct a dynamical system on the space Q^G. We call the elements of Q^G *configurations* as usual. In this way we obtain a cellular automaton on hyperbolic tessellation. We refer such automata as "tesselletion automata on free groups." In this paper we clarify relations among period preservability, injectivity and surjectivity of parallel maps. We also show the equivalence of finite orderedness and strong Poisson stability.

Historically, tessellation automata theory began with the work of Von Neumann [7]. Then Moore [6] showed the *Garden of Eden* theorem which states that violation of local injectivity implies existence of a Garden of Eden pattern. A Garden of Eden pattern is a partial configuration which cannot be reproduced in any environments. This shows an obstruction to *self-reproducing* property. Amoroso, Cooper and Patt [1] clarified the concept of a *Garden of Eden configuration.* Sato and Honda [8] investigated the relations among period preservability, Poisson stability and finite orderedness of parallel maps based on dynamical system theory. All these works, being very fruitful, were done in the framework of Euclidean tessellations. The aim of this paper is to extend cellular automata theory to non-Euclidean tessellations.

In section 1 we define tessellation automata on free groups and introduce group actions on them. In section 2 we define *periods* of configurations by using lattice of subgroups. In section 3 we state and prove the main theorems on injectivity, surjectivity, Poisson stability, strong Poisson stability and period preservability of parallel maps. In section 4 we state and prove the main theorem on finite orderedness and Poisson stability.