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Invariant nuclear space of a second quantization operator

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Abstract. Let $S'(\mathbf{R})$ be the dual of the Schwartz space $S(\mathbf{R})$, $\mathbf{A}$ a self-adjoint operator in $L^2(\mathbf{R})$ and $d\Gamma(\mathbf{A})^*$ the adjoint operator of $d\Gamma(\mathbf{A})$ which is the second quantization operator of $\mathbf{A}$. It is proven that under a suitable condition on $\mathbf{A}$ there exists a nuclear subspace S of a fundamental space $S_{\mathbf{A}}$ of Hida's type on $S'(\mathbf{R})$ such that $d\Gamma(\mathbf{A})S \subset S$ and $e^{-td\Gamma(\mathbf{A})}S \subset S$, which enables us to show that a stochastic differential equation arising from the central limit theorem for spatially extended neurons:

$$dX(t) = dW(t) - d\Gamma(\mathbf{A})^* X(t)dt,$$

has a unique solution on the dual space S' of S, where $W(t)$ is an S'-valued Wiener process.

1. Introduction

Concerning with infinite dimensional geometry and analysis, several types of fundamental spaces on infinite dimensional topological vector spaces have attracted several authors ([1], [4], [9], [11], [13], [14]). As it has been known by [5], the nuclearity of the space gives us the regularization theorem which guarantees the existence of a strong solution of the stochastic differential equation. However, [7] tried to construct a unique weak solution of a Segal-Langevin type stochastic differential equation on a suitable space of infinite dimensional generalized functionals which is not nuclear, and the fundamental spaces used in the Malliavin calculus are known not to be nuclear [2]. With this background, we consider spaces of Hida's type which are nuclear.

Let $(S_{\mathbf{A}})$ be a fundamental space of Hida's type and $d\Gamma(\mathbf{A})$ the second quantization operator. Inspired by the works [11], [12], we construct a fundamental space which is invariant under the semi-group $e^{-td\Gamma(\mathbf{A})}$ and is nuclear and smaller than $(S_{\mathbf{A}})$ even if $(S_{\mathbf{A}})$ is not nuclear. This enables us to obtain a unique strong solution of the stochastic differential equation

$$dX(t) = dW(t) - d\Gamma(\mathbf{A})^* X(t)dt, \tag{1.1}$$