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# The polynomials on $w_1$, $w_2$ and $w_3$ in the universal Wu classes

*Dedicated to Professor Teiichi Kobayashi on his 60th birthday*

Toshio YOSHIDA

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ABSTRACT. The cohomology ring $H^*(BO; Z_2)$ is the polynomial algebra $Z_2[w_1, w_2, w_3, \cdots]$, where $w_i$ is the $i$-dimensional universal Stiefel–Whitney class. The $i$-dimensional universal Wu class $v_i$ is defined inductively as follows: $v_0 = w_0 = 1$ and $w_i = v_i + \sum_{j=1}^{i} Sq^j v_{i-j} (i \geq 1)$, where $Sq^j$ is the Steenrod squaring operation. We can describe explicitly the polynomials on $w_1$, $w_2$ and $w_3$ in $v_i$.

## 1. Introduction

Let $BO$ be the space which classifies stable real vector bundles. Then its mod 2 cohomology $H^*(BO; Z_2)$ is the polynomial algebra over $Z_2$ on the universal Stiefel–Whitney classes $w_i \in H^i(BO; Z_2)$ for $i \geq 1$ (cf. [4], [10]).

The $i$-dimensional universal Wu class $v_i$ $(i \geq 0)$ is the element of $H^i(BO; Z_2)$, and this is defined inductively by using the Steenrod squaring operations $Sq^j$ in the following way (cf. [3], [6], [7], [8]):

(1.1) $\quad v_0 = w_0 = 1$ and $w_i = v_i + Sq^1 v_{i-1} + \cdots + Sq^i v_0 \quad$ if $\; i \geq 1$.

The $i$-dimensional Wu class $v_i(M)$ of a closed $n$-dimensional manifold $M$ is the unique element of $H^i(M; Z_2)$ such that

$$Sq^i x = x v_i(M) \qquad \text{for all} \quad x \in H^{n-i}(M; Z_2),$$

and the following relations between the Stiefel–Whitney classes and the Wu classes of $M$ hold (cf. [4], [9]):

(1.2) $\quad v_0(M) = 1$ and $w_i(M) = v_i(M) + Sq^1 v_{i-1}(M) + \cdots + Sq^i v_0(M) \quad$ if $i \geq 1$.

So if $f$ denotes the classifying map for the stable tangent bundle of $M$, then

$$f^* w_i = w_i(M) \text{ and } f^* v_i = v_i(M) \qquad \text{if} \quad i \geq 0.$$

Let $J$ be the ideal of $H^*(BO; Z_2)$ generated by the squares $w_1^2$, $w_2^2$, $w_3^2$, $\cdots$.

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