Statistical inference of Langevin distribution for directional data

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ABSTRACT. Comparison of some estimators and multi-sample tests about mean directions for the Langevin distribution have been studied. Before displaying main results, the background of directional statistics is briefly considered.

We derive the expectations and MSEs (mean square errors) of the MLEs (Maximum Likelihood estimators) of concentration parameter, κ and mean direction, μ in the forms of asymptotic expansions. We also compare the marginal MLE of κ with the MLE. It is shown that the estimators so modified as to satisfy a higher order asymptotic unbiasedness are the same in a higher order asymptotic sense. Further, it is shown that those estimators have smaller MSEs than the original ones when κ is not so small, but for small κ the MLE is preferable.

Next we consider some multi-sample tests for mean directions, μ_i's. Two cases are studied in detail. Namely, all μ_i's are on the same but unknown axis and μ_i's are in the given subspace.

1. Introduction

This paper is concerned primarily with the Langevin distribution for directional data. In general, there are three basic approaches to directional statistics, which are called embedding, wrapping and intrinsic approaches. For a discussion of these approaches, see Jupp and Mardia [1989]. They are usually used in different area, depending on their merits. In inferential problems, the embedding approach is commonly used, i.e., we consider to embed $(p-1)$-dimensional sphere $\mathscr{S}^{p-1}$ into p-dimensional Euclidean space $\mathscr{R}^p$. We also discuss a little more about this topic in the last section.

In section 2, we summarize some backgrounds of directional statistics, especially related to the Langevin distribution with a little refinement or improvement. Most of them are found in Mardia [1972] and Watson [1983a]. Therefore we give only an outline without strict proofs. Those are used in subsequent sections and will be helpful in understanding the rest of this paper. Although we do not put importance on applications here, Fisher et al. [1987] and Fisher [1993] have given a lot of examples.