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The chromatic E_1-term $H^1M_1^1$ at the prime 3

Yoshiko ARITA and Katsumi SHIMOMURA

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ABSTRACT. In this paper, we determine the E_1-term $H^1M_1^1$ of the chromatic spectral sequence converging to the E_2-term of the Adams-Novikov spectral sequence converging to the homotopy groups $\pi_*(M)$ of the mod 3 Moore spectrum M. At the prime $p > 3$, the E_1-term $H^1M_1^1$ plays a central role determining the homotopy groups $\pi_*(L_2M)$ of the $v_2^{-1}BP$-localized mod p Moore spectrum.

1. Introduction

Let M denote the mod p Moore spectrum and L_n the Bousfield localization functor with respect to $v_n^{-1}BP$. Here BP is the Brown-Peterson ring spectrum at a prime number p and v_n $(n = 1, 2, \dots)$ denotes the generator of $\pi_*(BP)$ with $|v_n| = 2p^n - 2$. Consider the spectrum N^1 obtained as a cofiber of the localization map $M \to L_1M$. In [12] and [9] H. Tamura and the second author determined the homotopy groups $\pi_*(L_2N^1)$ by using the Adams-Novikov spectral sequence at the prime $p > 3$. For $p > 3$ the Adams-Novikov filtration is at most 4 and the homotopy groups of L_2N^1 is determined by E_2-term [9]. At the prime $p = 3$, on the other hand, it is known that for any large integer $s_0 > 0$ there exists an integer $s > s_0$ such that the E_2-term $E_2^{s,*} \neq 0$ by the Morava structure theorem [8, Th. 6.2.10 (c)].

In this paper we will determine the first line of the E_2-term of the Adams-Novikov spectral sequence converging to $\pi_*(L_2N^1)$ at the prime 3. The E_2-term is an Ext group $\mathrm{Ext}^*_{BP_*(BP)}(BP_*, M_1^1)$ for a $BP_*(BP)$-comodule $BP_*(L_2N^1) = M_1^1$ which will be denoted by $H^*M_1^1$ following the paper on chromatic spectral sequences due to Miller, Ravenel and Wilson [6].

In order to state the result, we define integers $a(n)$, $a'(n)$ and a_n for $n \geq 0$ by:

$$a(0) = 2 \quad \text{and} \quad a(n) = 6 \cdot 3^{n-1} + 1 \quad (n > 0);$$

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