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# 3-primary $\beta$-family in stable homotopy of a finite spectrum

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ABSTRACT. Different from the case for a prime $p > 3$, the $\beta$-element $\beta_t \in \pi_{16t-6}(S^0)$ is not defined for each positive integer $t$ at the prime 3. Consider the 8-skeleton $X$ of the Brown-Peterson spectrum $BP$. Then we will show here that the $\beta$-element $\bar{\beta}_t \in \pi_{16t-6}(X)$ is defined for any positive integer $t$ even at the prime 3, and that they are all essential. These $\beta$-elements are obtained from $v_2$-maps on type 2 spectra. We use here $V(1) \wedge X$ as a type 2 spectrum instead of the Toda-Smith spectrum $V(1)$ that is used in the case $p > 3$.

## 1. Introduction

For each prime number $p$, a $p$-local finite spectrum $X$ is said to have type $n$ if $K(n)_*(X) \neq 0$ and $K(n-1)_*(X) = 0$ for the Morava $K$-theories $K(i)_*(-)$ with coefficient ring $K(i)_*(S^0) = \boldsymbol{Z}/p[v_i, v_i^{-1}]$. A self-map $\varphi : \Sigma^k X \to X$ of a $p$-local finite spectrum $X$ for some $k > 0$ is called $v_n$-map if $K(n)_*(\varphi) \neq 0$ and $K(m)_*(\varphi) = 0$ for $m \neq n$. M. Hopkins and J. Smith [1] show the existence of a $v_n$-map for every spectrum of type $n$, say, $\varphi : \Sigma^k X \to X$. Note that a $v_n$-map determines an integer $l > 0$ such that $K(n)_*(\varphi) = v_n^l$, and we cannot tell anything about $l$ from Hopkins-Smith's theorem. For $n = 1$ and $p > 2$, Adams gives a $v_1$-map $\alpha : \Sigma^{2p-2} V(0) \to V(0)$ with $l = 1$, where $V(0)$ denotes the mod $p$ Moore spectrum. Let $V(1)$ denote the cofiber of $\alpha$. Then it is a spectrum of type 2. For $n = 2$ and $p > 3$, L. Smith [9] show the existence of the $v_2$-map $\beta : \Sigma^{2p^2-2} V(1) \to V(1)$ with $l = 1$. These maps $\alpha$ and $\beta$ are used to define homotopy elements known as $\alpha$- and $\beta$-families in the stable homotopy groups of spheres.

Now we restrict our attention to the prime 3. Then Toda shows the non-existence of $v_2$-map on $V(1)$ with $l = 1$. So defining the $\beta$-family is a different story from the case $p > 3$, while $\beta$-family is given even at the prime 3. For example, $\beta_4$ does not exist at the prime 3. Recently, S. Pemmaraju [5] shows the existence of a $v_2$-map on $V(1)$ with $l = 9$. In this paper, we shall

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