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A note on pseudo resolvents

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ABSTRACT. Let $E \neq \{0\}$ be a Banach lattice. From elementary operator theory we know that for any bounded operator $\mathbf{T}$ mapping E into itself, the resolvent $\mathbf{R}(\lambda, \mathbf{T})$ of $\mathbf{T}$ satisfies the resolvent equation $\mathbf{R}(\lambda, \mathbf{T}) - \mathbf{R}(\mu, \mathbf{T}) = (\mu - \lambda)\mathbf{R}(\lambda, \mathbf{T})\mathbf{R}(\mu, \mathbf{T})$. The converse of the above statement in general is not true. In this note, we study the natural inverse problem. We investigate under what conditions a pseudo resolvent on E is the resolvent of a uniquely determined positive operator on E. Furthermore, we determine necessary and sufficient conditions for a pseudo resolvent to be the resolvent of a uniquely defined positive irreducible operator.

1 Introduction

In this note we provide necessary and sufficient conditions for a family of bounded operators satisfying the resolvent equation to be the resolvent of a uniquely defined positive irreducible operator on the Banach lattice. In the following we briefly summarize basic concepts and fundamental results.

Let $E \neq \{0\}$ be a Banach lattice, the subset $E_+ := \{x \in E | x \geqslant 0\}$ is called the *positive cone* of E, elements $x \in E_+$ are called *positive*, and any nontrivial element $x \in E_+$ will be denoted by the notation $x > 0$. A linear operator $\mathbf{S}$ mapping E into itself is called *positive* if $\mathbf{S}(E_+) \subset E_+$. We use $L(E)$ to denote the Banach space of all bounded linear operators mapping E into itself. A subset A of E is *solid* if $|x| \leqslant |y|$, and $y \in A$, implies $x \in A$. A solid subspace is called an *ideal*. A *principal ideal* is an ideal generated by a single element x and is denoted by E_x. It can be shown that if $x > 0$, then $E_x = \bigcup_{n=1}^{\infty} n[-x, x]$. Any $x \geqslant 0$ is called a *quasi-interior positive element* of E if its principal ideal E_x is dense in E, i.e., $\overline{E}_x = E$. A linear operator $\mathbf{S}: E \to E$ is called *ideal irreducible* if $\{0\}$ and E are the only $\mathbf{S}$-invariant closed ideals. We let $r(\mathbf{S})$ be the spectral radius of $\mathbf{S}$.

Let D be a nonempty open subset of $\boldsymbol{C}$, and let $R: D \to L(E)$ be a function satisfying

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