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Estimation and model selection in an extended growth curve model

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ABSTRACT. In this paper we consider the growth curve model with two different within-individuals design matrices. The MLE's of the model are obtained in closed forms, based on a canonical form. Some basic properties of the MLE's are given. The problem of selecting such a model is also considered. We derive a corrected version of *AIC* (Akaike Information Criterion, Akaike (1973)) which will be useful in small samples.

1. Introduction

An extension of the usual growth curve model introduced by Potthoff and Roy (1964) has been proposed by Verbyla and Venable (1988), who considered the model with several different within-individuals design matrices. In this paper we consider the growth curve model with two different within-individuals design matrices as follows: Let Y denote an $n \times p$ data matrix whose rows consist of observations on distinct experimental units. We assume that

$$Y = A_1\Theta_1 X_1 + A_2\Theta_2 X_{(2)} + \mathcal{E}, \tag{1.1}$$

where A_i are $n \times k_i$ between-individuals design matrices of ranks k_i, X_1 and $X_{(2)}$ are $q_1 \times p$ and $q \times p$ $(q = q_1 + q_2 \leq p)$ within-individuals design matrices of ranks q_1 and q, respectively, Θ_1 and Θ_2 are $k_1 \times q_1$ and $k_2 \times q$ matrices of unknown parameters and the rows of $\mathcal{E}$ are independently distributed, each with a p-variates normal distribution having mean zero and unknown covariance matrix Σ. Further, it is assumed that

$$X_{(2)} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \tag{1.2}$$

It may be noted that our model can be applied for the case where the n individuals have been measured at the same p different times or occasions, and consist of two types of polynomial growth curves with different degrees. In a polynomial growth curve model the design matrices within individuals are

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