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Decomposability of the mod p Whitehead element

Dedicated to Professor Yasutoshi Nomura on his 60th birthday

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ABSTRACT. We give necessary and sufficient conditions for the mod p Whitehead element $w_n \in \pi_{2np-3}(S^{2n-1})$ to be represented as a composition of some elements of positive stems in the homotopy groups of spheres. We also give necessary and sufficient conditions for w_n to be represented as Toda bracket of some elements of positive stems in the homotopy groups of spheres. Our problem is the odd primary version of the one studied by Iriye and Morisugi who treated the Whitehead product $[\iota_{2n-1}, \iota_{2n-1}]$ for the identity map ι_{2n-1} of S^{2n-1}.

1. Introduction

Let p be a fixed prime. In this paper we always assume that spaces are all localized at p. We study the decomposability of the mod p Whitehead element $w_n \in \pi_{2np-3}(S^{2n-1})$. Let $C(n)$ be the homotopy fiber of the double suspension $\Sigma^2 : S^{2n-1} \to \Omega^2 S^{2n+1}$, and $\varepsilon : C(n) \to S^{2n-1}$ the inclusion of the fiber. Then, it is known that $C(n)$ is $(2np-4)$-connected and $\pi_{2np-3}(C(n)) \cong \boldsymbol{Z}/p$. We denote a generator of $\pi_{2np-3}(C(n))$ by z. Then, according to the terminology due to [2], the mod p Whitehead element w_n is defined as $w_n = \varepsilon_*(z) \in \pi_{2np-3}(S^{2n-1})$. We will be concerned with w_n for an odd prime p.

Our main results are stated as follows; let $\alpha_i \in \pi^S_{2i(p-1)-1} \cong \boldsymbol{Z}/p$ for $i = 1, 2$ be a generator and $\langle -, -, - \rangle$ denote the Toda bracket [7].

THEOREM A. *Let p be an odd prime and $n \geq 2$. Then, w_n is decomposed as $w_n = \sum_i a_i b_i$ for some elements $\{a_i, b_i\}$ of positive stems in the homotopy groups of spheres if and only if one of the following holds*:

(1) *p is odd and $n = 2$, for which $w_n = \alpha_1\alpha_1$;*

(2) *$p = 3$ and $n = 3$, for which $w_n = \alpha_1\alpha_2$.*

THEOREM B. *Let p be an odd prime and $n \geq 2$. Then, w_n is represented as $w_n \in \sum_i \langle a_i, b_i, c_i \rangle$ for some elements $\{a_i, b_i, c_i\}$ of positive stems in the*

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