HIROSHIMA MATH. J.
26 (1996), 573–585

Linearized oscillations for neutral equations II: Even order

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(Received January 31, 1994)
(Revised May 29, 1995)

ABSTRACT. We consider a nonlinear neutral delay differential equation of the form $\frac{d^n}{dt^n}[x(t) - p(t)g((t-\tau))] = q(t)h(x(t-\delta))$, $t \geq t_0$ with $p(t)$, $q(t)$ continuous, $\tau > 0$, $\delta \geq 0$ and n even. We obtain sufficient conditions for oscillation of all bounded solutions for the case when $p(t)$ takes values outside $(0,1)$, and thereby establish some criteria as proposed in an earlier open problem.

1. Introduction

In this paper we consider the nonlinear neutral delay differential equation

$$\frac{d^n}{dt^n}[x(t) - p(t)g(x(t-\tau))] = q(t)h(x(t-\delta)), \qquad t \geq t_o \tag{1.1}$$

where n is an even integer,

$$p, q \in C([t_0, \infty), \boldsymbol{R}), \qquad g, h \in C(\boldsymbol{R}, \boldsymbol{R}), \qquad \tau > 0 \qquad \text{and} \qquad \delta \geq 0. \tag{1.2}$$

Recently, the linearized oscillation theory for nonlinear neutral delay differential equations has been extensively developed, for example see [1–3, 5–10]; in particular, [3] deals with the case when n is odd. Roughly speaking, it has been proved that, under appropriate hypotheses, certain nonlinear neutral delay differential equations have the same oscillatory character as an associated linear equation. The following linearized oscillation result for the equation (1.1) was obtained in [7] (see also [5]):

THEOREM A. *Assume that* (1.2) *holds,*

$$\limsup_{t\to\infty} p(t) = P_0 \in (0,1), \qquad \liminf_{t\to\infty} p(t) = p_0 \in (0,1),$$

$$\lim_{t\to\infty} q(t) = q_0 \in (0,\infty), \tag{1.3}$$

1991 *Mathematics Subject Classification.* Primary 34K40, Secondary 34C10.

Key words and phrases. Oscillation, neutral equation, linearized equation.

Research supported by NSERC-Canada and NNSF of China. Research was done when the second author was visiting the University of Alberta.