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On equilibrium solutions of a bistable reaction-diffusion equation with a nonlocal convection

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ABSTRACT. We are concerned with an ecological model described by a bistable reaction-diffusion equation with a nonlocal convection. We prove that there exist two symmetric stationary solutions.

1. Introduction

We are concerned with a bistable reaction-diffusion equation with a nonlocal convection.

$$u_t = du_{xx} - [(K * u)u]_x + ku(1-u)(u-a), \qquad x \in \boldsymbol{R},\ t > 0,\ (0 < a < 1) \tag{1}$$

where $K * u = \int_{\boldsymbol{R}} K(x-y)u(y,t)dy$ and d, k are positive constants. Here $u = u(x,t)$ denotes the population density at time t and the position x. The convection term $[(K * u)u]_x$ corresponds to an aggregating mechanism of the population. The function $f(u) = ku(1-u)(u-a)$ represents the growth rate of the population. Several spatially aggregating population models were discussed in [1], [2], [3] and [4]. In this paper we specify the kernel $K(x)$ to be the following function:

$$K(x) = \begin{cases} b & (x < 0) \\ -b & (x > 0), \end{cases} \tag{2}$$

where b is a positive constant. With this choice of the kernel, one can see that $u(x,t)$ moves in the right (resp. left) direction when

$$\int_{-\infty}^{x} u(y,t)dy - \int_{x}^{\infty} u(y,t)dy < 0. \qquad (\text{resp. } > 0)$$

This means that the individuals move in the direction of higher distribution. When the aggregative convection term $[(K * u)u]_x$ is absent, we have a well-known bistable reaction-diffusion equation.

$$u_t = du_{xx} + ku(1-u)(u-a).$$

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