Nonlinear perturbations of a class of integrated semigroups

Dedicated to Professor Hiroki Tanabe on the occasion of his sixtieth birthday

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ABSTRACT. Nonlinear perturbations of once integrated semigroups are treated in terms of nonlinear semigroup theory. Given an integrated semigroup $W(t)$ with generator A in a Banach space X, two general classes of nonlinear perturbations of the form $A + B$ are introduced. In order to define local quasidissipativity of $A + B$ and restrict the growth of solutions of the associated semilinear evolution equation (SE): $u'(t) = (A + B)u(t)$, a lower semicontinuous convex functional $\varphi: X \to [0, \infty]$ is employed. Necessary and sufficient conditions are given for a semilinear operator $A + B$ to generate a nonlinear semigroup $S(t)$ in X such that for $v \in D(B)$ the X-valued function $S(\cdot)v$ gives a unique weak solution. Application of the first main result to age-dependent population dynamics is discussed.

0. Introduction

The present paper is concerned with nonlinear semigroups which provide weak solutions to the semilinear problems of the form

$$\text{(SP)} \qquad \frac{d}{dt} u(t) = (A + B)u(t), \quad t > 0; \quad u(0) = v$$

in a real Banach space $(X, |\cdot|)$. Here A is assumed to be the generator of an integrated semigroup $\{W(t): t \geq 0\}$ in X and B is a nonlinear operator from a convex subset C of X into X.

The importance of semilinear problems of the type (SP) has constantly been recognized for many years in various branches of mathematical analysis. In this paper we introduce two general classes of nonlinear perturbations of linear integrated semigroups and discuss necessary and sufficient conditions on $A + B$ for the solutions of (SP) to exist in a global sense.