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# Immersions and embeddings of orientable manifolds up to unoriented cobordism

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ABSTRACT. We investigate the existence of immersions and embeddings of orientable manifolds in the Euclidean space up to unoriented cobordism, and we get the best estimates in some cases. Our study is an orientable version of the work investigated by R. L. Brown.

## 1. Introduction

In this paper, we investigate immersions and embeddings of orientable closed manifolds in the Euclidean space $\mathbf{R}^m$ up to unoriented cobordism. Manifolds are always assumed to be $C^\infty$ differentiable, and two closed $n$-dimensional manifolds $M_1^n$ and $M_2^n$ are *cobordant* if there exists a compact manifold $N^{n+1}$ whose boundary $\partial N$ is the disjoint union of $M_1$ and $M_2$. We refer to a closed manifold simply as a manifold.

The source of our study is the next theorem by Brown [1]:

THEOREM 1.1 (Brown). *Let $\alpha(n)$ denote the number of 1 in the dyadic expansion of $n$.*

(1) *For $n \geq 2$, any manifold $M^n$ is cobordant to a manifold which immerses in $\mathbf{R}^{2n-\alpha(n)}$ and embeds in $\mathbf{R}^{2n-\alpha(n)+1}$.*

(2) *For each $n \geq 2$ with $n \neq 3$, there is an $n$-dimensional manifold such that any manifold cobordant to it does not immerse in $\mathbf{R}^{2n-\alpha(n)-1}$ and does not embed in $\mathbf{R}^{2n-\alpha(n)}$.*

Our main results are stated as follows:

THEOREM A. *Let $\beta(n) = 2n - \alpha(n) - \min\{\alpha(n), \nu(n)\}$, where $\nu(n)$ is the integer determined by $n = (2m+1)2^{\nu(n)}$.*

(1) *Any orientable manifold $M^n$ is cobordant to a manifold which immerses in $\mathbf{R}^{\beta(n)}$ and embeds in $\mathbf{R}^{\beta(n)+1}$ for $n \geq 4$.*

(2) *If $n$ satisfies one of the following conditions* (i)–(iii), *then there is an*

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