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Nonexistence of positive solutions of Neumann problems for elliptic inequalities of the mean curvature type

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ABSTRACT. Exterior Neumann problems for quasilinear elliptic inequalities are considered. The leading terms of operators under consideration are the mean curvature type and generalized mean curvature types. Sufficient conditions are given for some Neumann problems to have no positive solutions.

0. Introduction

This paper concerns to elliptic boundary value problems of the form

$$\begin{cases} Mu \equiv \operatorname{div}\left[\dfrac{Du}{(1+|Du|^2)^\alpha}\right] \geq p(x)f(u), & x \in \Omega, \\ D_\nu u \leq 0, & x \in \partial\Omega, \end{cases} \tag{P}$$

where $x = (x_i)$, $Du = (D_i u)$, $D_i u = \partial u/\partial x_i$ for $i = 1, 2, \dots, N$, $N \geq 2$, $\Omega \subset \boldsymbol{R}^N$ is an exterior domain whose boundary $\partial\Omega$ is of class C^2, $\nu : \partial\Omega \to \boldsymbol{R}^N$, is a vector field pointing outward with respect to Ω, and $D_\nu u$ denotes the derivative of u along the vector ν. Throughout the paper we always assume the following without further mention:

(A_1) $0 \leq \alpha \leq 1/2$;
(A_2) $p : \bar{\Omega} \to (0, \infty)$ is continuous;
(A_3) $f : (0, \infty) \to (0, \infty)$ is locally Lipschitz continuous and strictly increasing with $\lim_{u\to\infty} f(u) = \infty$.

A typical example of f satisfying (A_3) is the function $f(u) = u^\sigma$, $\sigma > 0$. In this case we shall refer to (P) as (P_σ):

$$\begin{cases} Mu \geq p(x)u^\sigma, & x \in \Omega, \\ D_\nu u \leq 0, & x \in \partial\Omega. \end{cases} \tag{P_σ}$$

As is well known, when $\alpha = 1/2$, the operator M is called the mean

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