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Symmetricity of the Whitehead element

Dedicated to Professor Teiichi Kobayashi on his 60th birthday

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ABSTRACT. We study the symmetricity of the Whitehead element $w_n \in \pi_{2np-3}(S^{2n-1})$ for an odd prime p. It is shown that w_n considered as a map $S^{2np-3} \to S^{2n-1}$ factors through the p-fold covering map $\sigma: S^{2np-3} \to L^{2np-3}$ only when n is a power of p, and that w_{p^i} actually factors through σ if $0 \leq i \leq 4$. This is some of an odd prime version of the results of Randall and Lin for the projectivity of the Whitehead product $[\iota_{2n-1}, \iota_{2n-1}] \in \pi_{4n-3}(S^{2n-1})$.

1. Introduction

Let p be a prime, and $\sigma: S^{2n+1} \to L^{2n+1}$ denote the p-fold covering, where $L^{2n+1} = S^{2n+1}/\boldsymbol{Z}_p$ is the standard lens space. For any space X, an element $\alpha \in \pi_{2n+1}(X)$ is defined to be *symmetric*, if α considered as a map $S^{2n+1} \to X$ factors through $\sigma: S^{2n+1} \to L^{2n+1}$, that is, there exists a map $g: L^{2n+1} \to X$ with $\alpha = [g\sigma]$. Mimura-Mukai-Nishida [8] have shown that all elements in the positive dimensional stable homotopy groups of spheres are symmetric.

In this paper, we study the symmetricity of the Whitehead element $w_n \in \pi_{2np-3}(S^{2n-1})$ for an odd prime p. Hence, all spaces are assumed to be localized at an odd prime p. We recall the definition of w_n (cf. [3], [4]). Let $\varepsilon: C(n) \to S^{2n-1}$ be the homotopy fiber of the double suspension map $\Sigma^2: S^{2n-1} \to \Omega^2 S^{2n+1}$. It is known that $C(n)$ is $(2np-4)$-connected and $\pi_{2np-3}(C(n)) \cong \boldsymbol{Z}_p$. For a generator $z \in \pi_{2np-3}(C(n))$, w_n is given by $w_n = \varepsilon_*(z) \in \pi_{2np-3}(S^{2n-1})$. Then, our results are stated as follows:

THEOREM A. *If the Whitehead element $w_n \in \pi_{2np-3}(S^{2n-1})$ is symmetric, then $n = p^i$ for some $i \geq 0$.*

THEOREM B. *The Whitehead element $w_{p^i} \in \pi_{2p^{i+1}-3}(S^{2p^i-1})$ is symmetric for $0 \leq i \leq 4$.*

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