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Oscillation and nonoscillation theorems for a class of second order quasilinear functional differential equations

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ABSTRACT. The equation to be studied in this paper is

$$(A) \qquad (|y'(t)|^{\alpha-1}y'(t))' + f(t, y(g(t))) = 0, \qquad \alpha > 0.$$

Under certain assumptions on f and g, classification of nonosillatory solutions of (A) is given according to their asymptotic behavior as $t \to \infty$. Criteria are obtained for the existence and nonexistence of nonoscillatory solutions of (A). As a result one can indicate a class of equations of the form (A) for which the situation for oscillations of all solutions can be completely characterized.

0. Introduction

The purpose of this paper is to study the oscillatory and nonoscillatory behavior of quasilinear functional differential equations of the type

$$(A) \qquad (|y'(t)|^{\alpha-1}y'(t))' + f(t, y(g(t))) = 0$$

for which the following conditions, collectively referred to as (H), are assumed to hold:

(a) α is a positive constant;

(b) $g(t)$ is a positive continuous function on $[a, \infty)$, $a \geq 0$, such that $\lim_{t\to\infty} g(t) = \infty$;

(c) $f(t, y)$ is a continuous function on $[a, \infty) \times \mathbf{R}$ which is nondecreasing in y and satisfies $yf(t, y) > 0$, $y \neq 0$, for each fixed $t \geq a$.

By a solution of (A) we mean a function $y \in C^1[T_y, \infty), T_y \geq a$, which has the property that $|y'|^{\alpha-1}y' \in C^1[T_y, \infty)$ and satisfies the equation at all sufficiently large t in $[T_y, \infty)$. Our attention will be restricted to those solutions $y(t)$ of (A) which are nontrivial in the sense that $\sup\{|y(t)| : t \geq T\} > 0$ for any $T \geq T_y$. A solution is said to be oscillatory if it has an infinite sequence of zeros clustering at $t = \infty$; otherwise a solution is said to be nonoscillatory.

It can be shown that, as regards the asymptotic behavior of a nonoscillatory solution $y(t)$ of (A), the following three cases are possible:

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