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Strong solution for a mixed problem with nonlocal condition for certain pluriparabolic equations

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ABSTRACT. The present paper is devoted to a proof of the existence and uniqueness of a strong solution for a mixed problem with nonlocal condition for certain pluriparabolic equations. The proof is based on an a priori estimate and on the density of the range of the operator generated by the studied problem.

1. Statement of the problem

In the domain $Q = (0, b) \times (0, T_1) \times (0, T_2)$, with $b < \infty$, $T_1 < \infty$ and $T_2 < \infty$, we consider the one-dimensional pluriparabolic equation

$$\mathscr{L}v = \partial v/\partial t_1 + \partial v/\partial t_2 - \partial(a(x, t_1, t_2)\partial v/\partial x)/\partial x = \not{f}(x, t_1, t_2), \tag{1.1}$$

where $a(x, t_1, t_2)$ satisfy the following assumptions:

H1. $c_0 \leq a(x, t_1, t_2) \leq c_1$, $\partial a(x, t_1, t_2)/\partial x \leq c_2$, $\partial a(x, t_1, t_2)/\partial t_p \leq c_3$, $p = 1, 2$, $(x, t_1, t_2) \in \bar{Q}$.

H2. $\partial^2 a(x, t_1, t_2)/\partial t_p^2 \leq c_4$, $\partial^2 a(x, t_1, t_2)/\partial x^2 \leq c_5$, $\partial^2 a(x, t_1, t_2)/\partial t_p \partial x \leq c_6$, $p = 1, 2$, $(x, t_1, t_2) \in \bar{Q}$.

We pose the following problem for equation (1.1): to determine its solution v in Q satisfying the initial conditions

$$\ell_1 v = v(x, 0, t_2) = \Phi_1(x, t_2), \qquad (x, t_2) \in Q_2 = (0, b) \times (0, T_2), \tag{1.2}$$

$$\ell_2 v = v(x, t_1, 0) = \Phi_2(x, t_1), \qquad (x, t_1) \in Q_1 = (0, b) \times (0, T_1), \tag{1.3}$$

the Neumann condition

$$\partial v(0, t_1, t_2)/\partial x = \mu(t_1, t_2), \qquad (t_1, t_2) \in (0, T_1) \times (0, T_2), \tag{1.4}$$

and the integral condition

$$\int_0^b v(x, t_1, t_2)dx = E(t_1, t_2), \qquad (t_1, t_2) \in (0, T_1) \times (0, T_2). \tag{1.5}$$

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