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Invariance principle for a Brownian motion with large drift in a white noise environment

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ABSTRACT. This paper discusses an invariance principle for a Brownian motion with drift coefficient $\kappa/4$ in a white noise environment under the assumption that κ is large. Our method clarifies the relation between the environment-wise invariance principle discussed in [7] and the present result (the invariance principle in random environment).

Introduction

Let W be the space of continuous functions on $\mathbf{R}$ vanishing at 0 that is equipped with the Wiener measure P. For an element $w \in W$ let us denote by w_κ the element of W defined by $w_\kappa(x) = w(x) - (\kappa x/2)$ where κ is a given positive constant. For $w \in W$, P_w denotes the probability measure on $\Omega = C[0, \infty)$ such that $\mathbf{X}_x = \{\omega(t), t \geq 0, P_w\}$ is a diffusion process with generator

$$\mathscr{L}_w = \frac{1}{2} e^{w_\kappa}(x) \frac{d}{dx}\left(e^{-w_\kappa}(x) \frac{d}{dx}\right)$$

starting at 0, where $\omega(t)$ is the value of a function $\omega(\in \Omega)$ at time t. We regard $\omega(t)$ as a process defined on the probability space $\{W \times \Omega, \mathscr{P}\}$ where $\mathscr{P}(dw\, d\omega) = P(dw)P_w(d\omega)$. Then symbolically

$$d\omega(t) = dB(t) + \frac{\kappa}{4}\, dt - \frac{1}{2} w'(\omega(t))\, dt,$$

where $B(t)$ is a standard Brownian motion independent of the white noise $\{w'(x)\}$. We call the process $\mathbf{X} = \{\omega(t), t \geq 0, \mathscr{P}\}$ a Brownian motion with drift in a white noise environment; in [2] [6] [7] it is called a diffusion process in a Brownian environment with drift. The present authors obtained some limit theorems for $\mathbf{X}$ in [2] (see [8] for further results; see also [6] for a brief survey on related problems), which are analogous to those of [3] and [5]; however, some problems remain open. The present paper is a continuation of [7] and

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