HIROSHIMA MATH. J.
28 (1998), 1–5

A remark on homology localization

Tetsusuke OHKAWA

(Received January 20, 1994)
(Received December 27, 1996)

ABSTRACT. A. K. Bousfield [1], [2] introduced the notion of localization of spaces and spectra with respect to homology functor h and proved the exsitence theorem. In this note we introduce a variation of this notion, (h, n)-localization, which interpolates a contractible space or spectrum pt and the localization $L_h(X)$ of the original space or spectrum X and prove the existence theorem along the arguments of [1], [2].

1. Statement of results

Let $\mathscr{C}, \mathscr{S}, \tilde{\mathscr{C}}, \tilde{\mathscr{S}}$ denote the categories of CW-complexes, CW-spectra and their homotopy categories respectively.

DEFINITION. Let $\mathscr{A}, \mathscr{B}$ be categories and $\mathscr{F} : \mathscr{A} \to \mathscr{B}$ a functor.

i) $C \in Ob(\mathscr{A})$ is called *$\mathscr{F}$-local* if $f^* : \mathscr{A}(B, C) \to \mathscr{A}(A, C)$ is bijective for any $A, B \in Ob(\mathscr{A})$ and any $f : A \to B$ such that $\mathscr{F}(f)$ is an isomorphism.

ii) A morphism $g : A \to C$ is called an *$\mathscr{F}$-localization map* of A if C is $\mathscr{F}$-local and $\mathscr{F}(g)$ is an isomorphism. In this case C is called an *$\mathscr{F}$-localization* of A.

Let h be a generalized homology functor and n an integer. Let $\alpha = (h, n)$ be the functor defined by $\alpha(X) = (h_k(X) | k < n)$ from $\mathscr{D}(= \tilde{\mathscr{C}}$ *or* $\tilde{\mathscr{S}})$ to $\{(A_k | k < n); A_k \in Ab\}$, *where Ab is the category of abelian groups. Then we can prove the following.*

THEOREM 1. *Let h be a generalized homology functor which is representable by a spectrum and $\alpha = (h, n)$ the functor above for an integer n. Then it holds that*

i) *Any object $X \in Ob(\mathscr{D})$ has an α-localization map.*

ii) *Let $f : X \to Y$ and $g : X \to Z$ be α-localization maps of X. Then there exisits a map $k : Y \to Z$ such that $k \circ f \simeq g$. Moreover, such a map k is always an isomorphism in the category $\mathscr{D}$.*

Note that (h, n)-localization of X is unique up to homotopy by this theorem. This may be called also *half h-localization* and denoted by $L_h^n(X)$.