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Exponential integrability for Riesz potentials of functions in Orlicz classes

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Abstract. Our aim in this paper is to show the exponential integrability for Riesz potentials of functions in an Orlicz class. As a corollary, we show the double exponential integrability given by Edmunds-Gurka-Opic [3], [4].

1. Introduction

For $0 < \alpha < n$, we define the Riesz potential of order α for a nonnegative measurable function f on $\mathbf{R}^n$ by

$$R_\alpha f(x) = \int |x-y|^{\alpha-n} f(y)\, dy.$$

In this paper, we give the following theorems, which deal with the limiting cases of Sobolev's imbeddings.

Theorem A. *Let f be a nonnegative measurable function on a bounded open set $G \subset \mathbf{R}^n$ satisfying the Orlicz condition*

$$\int_G f(y)^p [\log(e+f(y))]^a [\log(e+\log(e+f(y)))]^b\, dy < \infty \tag{1.1}$$

for some numbers p, a and b. If $\alpha p = n$, $a < p-1$, $\beta = p/(p-1-a)$ and $\gamma = b/(p-1-a)$, then

$$\int_G \exp[A(R_\alpha f(x))^\beta (\log(e+R_\alpha f(x)))^\gamma]\, dx < \infty \qquad \textit{for any } A > 0. \tag{1.2}$$

In case $a = b = 0$, inequality (1.2) is well known to hold (see [1], [9], [12], [13]). The case $a < p-1$ and $b = 0$ was proved by Edmunds-Krbec [5] and Edmunds-Gurka-Opic [3], [4]; see also Brézis-Wainger [2].

In view of Theorem A, we see that (1.2) is true for every $\beta > 0$ (and $\gamma > 0$) when $a \geqq p-1$. In case $a > p-1$, we know that $R_\alpha f$ is continuous on $\mathbf{R}^n$ (see [7] and [10]).

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