HIROSHIMA MATH. J.
28 (1998), 337–344

Based modules and good filtrations in algebraic groups

Masaharu KANEDA

(Received May 12, 1997)

ABSTRACT. Let $\mathfrak{G}_{\mathfrak{k}}$ be a simply connected semisimple algebraic group over an algebraically closed field $\mathfrak{k}$ of positive characteristic with simple system of roots Π. After the initial efforts by Wang J.-P. and S. Donkin, O. Mathieu proved, using the Frobenius splitting of the flag variety, Donkin's conjectures that (i) if Π' is a subset of Π and if $\mathfrak{G}'_{\mathfrak{k}}$ is the semisimple subgroup of $\mathfrak{G}_{\mathfrak{k}}$ generated by the root subgroups associated to Π', then any Weyl module of $\mathfrak{G}_{\mathfrak{k}}$ admits a filtration by $\mathfrak{G}'_{\mathfrak{k}}$-modules all of whose subquotients are Weyl modules for $\mathfrak{G}'_{\mathfrak{k}}$; (ii) the tensor product of any two Weyl modules of $\mathfrak{G}_{\mathfrak{k}}$ admits a filtration by $\mathfrak{G}_{\mathfrak{k}}$-modules all of whose subquotients are Weyl modules of $\mathfrak{G}_{\mathfrak{k}}$. In this note we explain that the conjectures can also be obtained as immediate consequences of Lusztig's results on based modules.

Introduction

Let $\mathfrak{G}_{\mathfrak{k}}$ be a simply connected semisimple algebraic group over an algebraically closed field $\mathfrak{k}$ of positive characteristic with simple system of roots Π. After the initial efforts by Wang J.-P. and S. Donkin, using the Frobenius splitting of the flag variety O. Mathieu [M] proved Donkin's conjectures that (i) if Π' is a subset of Π and if $\mathfrak{G}'_{\mathfrak{k}}$ is the semisimple subgroup of $\mathfrak{G}_{\mathfrak{k}}$ generated by the root subgroups associated to Π', then any Weyl module of $\mathfrak{G}_{\mathfrak{k}}$ admits a filtration by $\mathfrak{G}'_{\mathfrak{k}}$-modules all of whose subquotients are Weyl modules for $\mathfrak{G}'_{\mathfrak{k}}$; (ii) the tensor product of any two Weyl modules of $\mathfrak{G}_{\mathfrak{k}}$ admits a filtration by $\mathfrak{G}_{\mathfrak{k}}$-modules all of whose subquotients are Weyl modules of $\mathfrak{G}_{\mathfrak{k}}$. Since then J. Paradowski [P] has given another proof using Lusztig's canonical basis. There is yet a third proof using Kashiwara's crystal base; Donkin's conjectures are immediate consequences of Lusztig's results on based modules [L], which may be worth pointing out after the appearence of a friendly account [J] of crystal bases. The third proof works naturally over $\boldsymbol{Z}$, hence over any commutative ring, and is free of Donkin's cohomological criterion for the existence of good filtrations [JG, II.4.16].

In §1 of the present note we will restate Lusztig's results in the framework of [J], and show Donkin's conjectures. We see that the proof is logically

1991 *Mathematics Subject Classification* 20G
Key words and phrases, Based modules, Crystal Bases, Good filtrations