HIROSHIMA MATH. J.
28 (1998), 321–327

On maps into a co-H-space

Marek GOLASIŃSKI and John R. KLEIN

(Received March 10, 1997)

ABSTRACT. We prove that the domain of a map $X \to Y$ to a co-H-space inherits a co-H-structure provided some dimensionality and connectivity properties hold. Then we deduce that a space X admits a co-H-structure if and only if on all its skeletons there is such a structure as well. Moreover, if X is 1-connected then a co-H-structure on X is equivalent to such a structure on all its homology decomposition stages.

Introduction.

Recall [5, Chapter IX], where it was shown that a map $X \to Y$ of based spaces extends an H-space structure on X to one on Y provided some connectivity properties hold. In particular, the result implies that an H-structure on X is inherited on all its Postnikov stages.

This paper examines the dual problem. However, the arguments used for H-spaces do not seem to dualize. Therefore, we make use of the one-to-one correspondence (see [1] or [2, p. 209–212]) between homotopy classes of co-H-structures on a space X and those of coretractions $X \to \Sigma\Omega X$. A result due to Hilton (see [2, p. 185]) says that if a space X is dominated by a co-H-space Y then X also admits a co-H-structure. We prove (Theorem 1) that the domain of a map $X \to Y$ to a co-H-space inherits a co-H-structure provided some dimensionality and connectivity properties hold. Then we deduce (Corollary 1) that a space X admits a co-H-structure if and only if on all its skeletons there is such a structure as well. Moreover, if X is 1-connected then Corollary 3 states that a co-H-structure on X is equivalent to such a structure on all its homology decomposition stages and a dualization of Corollary 5.6 in [5, page 443] is obtained provided X is 2-connected.

1. Preliminaries.

We consider based spaces of the based homotopy type of a based CW-complex; the basepoints are assumed to be non-degenerate. All maps and

1991 *Mathematics Subject Classification*. Primary 55P40, 55P45; secondary 18C15, 55P15.

Key words and phrases: co-H-space, co-H-equivalence, flat and smash product, homotopy pull-back, suspension and loop functor.