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Generalized functions in infinite dimensional analysis

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ABSTRACT. We give a general approach to infinite dimensional non-Gaussian Analysis which generalizes the work [2] to measures which possess more singular logarithmic derivative. This framework also includes the possibility to handle measures of Poisson type.

1. Background and Introduction

White Noise Analysis and—more generally—Gaussian analysis have now become of age, both date back approximately twenty years, for reviews we refer to [4, 13]. Essential to both of them is an orthogonal decomposition of the underlying L^2 space—the "chaos" or "Hermite" or "normal" or "multiple Wiener integral" decomposition.

One extension of this setup has been introduced by Y. M. Berezansky: Starting from certain field operators he constructs polynomial or orthogonal decompositions with respect to the spectrum measures which need not necessary be Gaussian, see e.g., [5].

A different approach was recently proposed by [1]. For smooth probability measures on infinite dimensional linear spaces a biorthogonal decomposition is a natural extension of the orthogonal one that is well known in Gaussian analysis. This biorthogonal "Appell" system has been constructed for smooth measures by Yu. L. Daletskii [8]. For a detailed description of its use in infinite dimensional analysis we refer to [2].

Aim of the present work. We consider the case of non-degenerate measures on co-nuclear spaces with analytic characteristic functionals. It is worth emphasizing that no further condition such as quasi-invariance of the measure or smoothness of logarithmic derivatives are required. The point here is that the important example of Poisson noise is now accessible.

For any such measure μ we construct and Appell system $\mathbb{A}^{\mu}$ as a pair $(\mathbb{P}^{\mu}, \mathbb{Q}^{\mu})$ of Appell polynomials $\mathbb{P}^{\mu}$ and a canonical system of generalized functions $\mathbb{Q}^{\mu}$, properly associated to the measure μ.

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